How long does it take to Beggar My Neighbour? Mike Mudge invites attempts at a computer-generated solution.*

*Oxford English Dictionary: *beggar*..v.t. Reduce to poverty.

The area of investigation this month arises from the card game sometimes known as 'Beggar My Neighbour' and has been suggested by Peter Walwyn of Porthmadog, Gwynedd.

Rules of Beggar My Neighbour
The game is played with one standard 52-card pack which, after shuffling, is dealt *face downwards* into two equal hands.

The toss of a coin may be used to determine which player (hand) starts the first game; after that, the winner of the previous game deals and starts the associated game.

The players hold their hands *face downwards* and take it in turns to remove the top card from their hand and place it *face upwards* on top of the pile. This process continues until either: (1) the player whose turn it is to play has no more cards, in which event the opponent is declared the winner; or (2) a face card (Ace value 4, King value 3, Queen value 2 or Jack value 1) is turned up and placed upon the pile, whereupon the other player has to 'pay' a number of cards from the top of the hand up to the above value.

If, however, in the course of paying, another face card is paid, then the opponent begins to pay in the same way until either: (a) the full value has been paid in consecutive non-face cards, in which case the non-payer picks up the entire pile, turns it over *face downwards*, places it under the hand, and then begins the next play; or (b) the paying player runs out of cards, in which case the opponent is declared the winner.

Project A Design and implement a computer program to play Beggar My Neighbour. Peter Walwyn uses GW-Basic compiled using Microsoft's Bascon v1.0 (1982) and run on an SBC HD20 XT with RND to simulate the dealing of the hands. The average run time achieved is 5.25 games per second.

Test Data A (. denotes a non-face card)
Player 1
..JAQ.....Q..JQ......J....
Top of hands.
Player 2
....QJ....KA....KA..K.K.A.
Player 1 starts and wins after 112 plays.

Test Data B
Player 1
K.Q..A.QA..K.........J.K.
Top of hands.
Player 2 Q.J..J.....AQ....K
Player 1 starts and wins after 3092 plays.

Project B Investigate this beggaring algorithm in depth. Construct an empirical statistical distribution of length of game (in plays) against frequency of occurrence.

How many possible distinct games are there? (Note that only the locations of the face cards are important.)

****** Are there hands which loop?****

(Peter asks if there exists a neat matrix formulation of the game to allow algebraic iteration of N-plays?)

Thought for the month
Personal computing is essentially a solitary occupation. Christmas can be a very lonely time. Computer software is readily available for playing chess, backgammon, bridge, and so on, not to mention the myriad of purpose designed computer games. What knowledge, experience or other thoughts do readers have regarding computer patience?

The software would of course replace the pack of cards, and should ensure that the rules are not broken. Supply, where possible, a statistical prediction of the possible outcome, and cover many versions of patience. *The Complete Patience Book* by Basil Dalton (entirely reset in 1964 by John Baker Publishers) describes 52 versions of patience selected from some 350 and seems a natural starting point for such a project.

Attempts at some or all of the above projects may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel 0974 272548 to arrive by 1 March 1991. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained; together with suggestions for further work in this area, all in a form suitable for publication in *PCW*.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for further Numbers Count articles.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Happy New Year to all! It's 1991, the only palindromic year this century (although 1961 reads the same upside down).

Here's a quickie to start the year off. No answers, no prizes. A domino set with the highest domino of double-six contains 28 pieces. How many dominoes would there be in a set going up to double-twelve?

Prize Puzzle
And now, as they say, for something completely different. A genuine simulation model required here.

For the uninitiated, a shove-ha'penny board is rather like a zebra crossing, with horizontal lines equally spaced along its length. If you were to throw a stick of length equal to the distance between two lines, randomly onto the board, it would obviously fall either across a line or clear of a line. (Assume that the board is of infinite width.)

For this month's puzzle, I want you to throw the stick (by computer, preferably, but manually if you wish) over and over again, and divide the number of occasions on which the stick crosses a line by the total number of throws. Multiply this ratio by 4 and send me the answer — you might be surprised at what you get, assuming you do sufficient throws.

A little bit of thought and Pythagoras (or trigonometry) will be needed at the outset to set up the model, but not much. The rest depends on the random number generator in your computer.

Answers on postcards or backs of sealed envelopes — no letters please. Send to: January Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 31 January 1991.

Winner, October 1990
The winning entry came from that most exotic of seaside resorts, Southend on Sea, from Mr Bruce Williams who gets our congratulations and, quite soon we hope, his prize.

The correct solution was: 4,383,592. A bit easy really, although this month's puzzle is rather more difficult. So, to the also-rans, don't give up — this month you could win.

NUMBERS COUNT

Got a problem with Babylonian fractions? Mike Mudge has five — see if you can solve them.

This month's area of investigation has been suggested by Dr Nigel Backhouse of Helsby, Warrington, Cheshire, who relates the problem to work done by the Babylonians some three and a half to four thousand years ago. The mathematical prerequisites are nothing other than the algebraic rules for manipulating fractions, but there appears to be considerable scope for ingenuity of programming. Nigel describes himself as 'by no means an expert in number theory, even less so with anything to do with computers! . . . but having a general interest in algorithms.'

The Babylonian notation for a fraction represented only those fractions of the form 1/N. Thus in order to use a more general fraction it had to be split up into a sum of such unit fractions. For example: $5/6 = 1/2 + 1/3$; $7/20 = 1/3 + 1/60$; $9/20 = 1/4 = 1/5$.

Problem 1 Given a proper fraction P/Q (that is, where P is less than Q and the pair P,Q have no common integer divisor) express it as a sum of unit fractions. *Note.* Ignore the trivial solution where 1/Q is added to itself P-times.

Possible algorithm for this problem
$F(0) = P/Q$,
$F(K+1) = F(K) -$ [the largest unit fraction less than or equal to $F(K)$.]
The sequence $F(0)$, $F(1)$, $F(2)$...tends to zero and so (1)...$P/Q = [F(0) - F(1)] + [F(1) - F(2)] + \ldots$gives a possible solution.

For example: $5/6 = 1/2 + 1/3$; $7/20 = 1/3 + 1/60$; $9/20 = 1/3 + 1.9 + 1/180$.

It is to be observed that the representation (1) above must be finite (that is, terminate) because the numerators of the fractions decrease strictly monotonically, and eventually unity must be reached. Thus at most P unit fractions are needed, this is sometimes the case, for example $2/3 = 1/2 + 1.6$.

Problem 2 Extension of Problem 1 above and its suggested algorithmic solution to any irrational number, lying between 0 and 1, to obtain a non-terminating representation as a sum of unit fractions. Can any use be suggested for this representation?

Problem 3 Given that for P/Q the expansion in unit fractions is not unique, *viz* $9/20 = 1.3 + 1/10 + 1/60 = 1/4 + 1/5$, and that further the above algorithm does not necessarily yield the shortest expansion: is there an algorithm which always yields the expansion with the smallest number of unit fractions?

Problem 4 How best does one obtain lists of expansions of P/5, P/6, P/7 and so on. Is there an underlying pattern? Does anything special happen when Q, the denominator throughout this work, is a prime number?

Possible Clue? If one has expansions of P/Q and R/S then one can immediately write down (some?) expansions of (PR)/(QS). Nigel suggests here that 'A large bank of examples might help', for example $(5/6)(7/20) = 7/24 = (1/2 + 1/3)(1/3 + 1/60) = 1/6 + 1/120 + 1/9 + 1/180$.

Problem 5 How does the theory of representation of irrational numbers, lying between 0 and 1, as non-terminating sums of unit fractions vary with number base used for the arithmetic? For example, how does the accuracy of a truncated representation depend upon the number base?

Attempts at some or all of the above problems may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel 0974-272548, to arrive by 1 April 1991. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained; together with suggestions for further work in this area, all in a form suitable for publication in *PCW*. Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, August 1990
The response to this article, the second of three parts due to Norman Meeres of 89 Grove Way, Esher, Surrey KT10 8HF, included a very detailed study by Frank Webster of Middlesborough. He examined project A with $p < 10,000$ in 65 minutes, project B with $p \leq 9967$ in 67 minutes, project C with $p < 10,000$ in 42 minutes and project D with $p < 20,000$ in 120 minutes. After much soul searching the prizewinner this month is Norman Meeres for his totality of effort divided over the three parts of this study.

As promised in *PCW* December 1990 an overview of the relationships between quadratic partitions and cubic residues can be expected in June 1991 when all relevant submissions have been analysed and a closing comment from Norman obtained.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for further Numbers Count articles.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie

Here's a February quickie for you — no prizes, no answers. When Albert and Ken ran a 100 yard race, Albert won by 4 yards. Next day they raced again, but this time Ken got 4 yards start. Assuming they both ran at the same speed as before, who would be the winner? (The answer is *not* a dead heat.)

Prize Puzzle

And now for a real number cruncher, although it shouldn't prove too difficult. There are three parts:

A certain number contains n digits, and the sum of the digits raised to the power n equals the number itself. To illustrate:

The 3-digit number
$153 = 1^3 + 5^3 + 3^3$

The 4-digit number
$1634 = 1^4 + 6^4 + 3^3 + 4^4$

Can you find a 5-digit, a 6-digit, and a 7-digit number with the above property *and* which between them use every digit from 0-9 except the digit 7?

Send your three answers on postcards or backs of sealed envelopes (no letters please) to February Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 28 February 1991.

Winner, November

The problem of the number of triangles created by joining every vertex of a regular octagon proved a much tougher puzzle than usual. Only 46 entries were received and half of those were incorrect.

The winning entry came from Chester, from Mr GE Reynolds. Well done, Mr Reynolds — your prize is on its way. The correct number of triangles was 632.

Don't give up — it could be your turn next. ∎

NUMBERS COUNT

Mike Mudge presents you with an opportunity to use your Random Number Generator as he poses a problem with dice.

This month's problem is based upon a suggestion by Neil Duncan of Ealing, London W5 2AD. It is the sort of problem that may be readily soluble by the technique of manipulating 'probability generating functions'. However, here it will be used to show how a simple probability situation can be easily generalised to generate inconveniently large numbers and disarmingly simple questions. It will also be used to illustrate the limitations of (micro)computers, namely their inability to think!

If player A throws a dice (assumed throughout to be an unbiased cubical dice numbered 1-6) and offers player B odds of 5-1 against getting the same score with one throw, this is clearly a fair bet. If nowever player B is permitted a second throw, and then to add the scores together if the first one is too low and then again if the total is still too low, and so on, so that B is trying to make a *total* score equal to the *single* score of A, would odds of 3-1 be fair? Or suppose that player A threw several dice simultaneously, with the *total* score the target for player B with no limit on the number of throws, would odds of 5-2 be fair?

Consider first the case where player A throws only one dice with scores of 1,2,3,4,5,6 all equally likely with probability 1/6. With a throw of 1 player B can win in one way only, *viz* by getting a 1 first throw, a 1/6 chance. However with a throw of 6 B can win in one way with one throw (6), in one way with six throws (1+1+1+1+1+1), in five ways with two throws (1+5,2+4,3+3,4+2,5+1) or with five permutations of 2,1,1,1,1 and in ten ways with either three or four throws. So, the overall chance of scoring six is given by 1/6 + 5/36 + 10/216 + 10/1296 + 5/7776 + 1/46656 which totals 16807/46656 or approximately 0.360232. This discussion covers the chances of B winning when A throws 1 or 6. A complete investigation averaging over all six results yields 70993/279936 or approximately 0.253604 so the suggested odds of 3-1 are essentially fair.

The same principles can be applied to player A throwing two or more dice but they soon become tedious. For example, with two dice scores from 2 to 12 are possible and the intermediate totals are not equally likely. Further, in response to a score of 12 there are 1936 ways for B to win, one of which (twelve ones) has a probability of only 1 in 2176782336. A complete analysis of this case has been carried out by Neil Duncan with a result of approximately 0.2835. Would the chance of B winning improve if A used more than two dice?

Let D denote the number of dice thrown by player A, recall that B continues to throw one dice until either winning (that is, equalling the total score of A) or losing (that is, exceeding the total score of A); the table below shows the probability, p, that B wins correct to nine decimal places.

The immediate question is: Does this probability tend to a limit as D increases? Almost self evident, is this limit equal to 2/7? If so, why, so if not, why not?

Problem 1 Construct a computer program to evaluate p as a function of D, wherever possible exact arithmetic should be used. The proposer observes that tabulation of the numbers of ways of scoring S with D dice soon reveals a pattern that can be produced by applying simple rules. 'If you teach the computer the rules, it will happily calculate your chance of B winning with *any* number of dice in use by A.'

Problem 2 Simulate this dice game using a random number generator and obtain empirical values of p as a function of D. How many trials are required to obtain a given accuracy in p? There is a suggestion that 100,000 trials give p correct to two decimal places!

Compare and contrast the two approaches to the limiting value of p.

Attempts at either or both of the above problems may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel 0974-272548, to arrive by 1 May 1991. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained; together with suggestions for further work in this area, all in a form suitable for publication in *PCW*.

Mike Mudge welcomes correspondence on any subject within the area of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.

D	1	2	7	8	12
p	0.253604395	0.283539658	0.285714342	0.285714282	0.285714284

LEISURE LINES

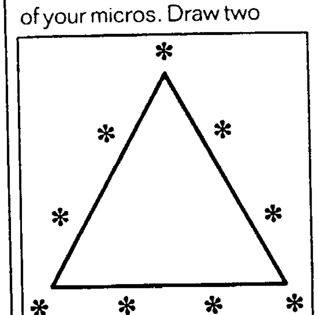
Brainteasers courtesy of JJ Clessa.

This Month's Quickie

A nifty quickie for March — no prizes, no answers. A hiker walks at 2mph uphill and 6mph downhill. He walks up a hill, turns straight round at the top, and walks back down. What is his average speed for the trip? (Don't say 4mph — it's the wrong answer.)

Prize Puzzle

And now for something different — a logic puzzle which you may be able to crack with or without the help of your micros. Draw two



equilateral triangles and place along the sides of each, nine digits in the positions shown above, so that:

1) No digit may appear twice on the same triangle.
2) The sum of the digits along any side of a triangle is 14.
3) All 10 digits (0-9) are used.

Which two digits appear only once?

Send your answers on postcards or backs of sealed envelopes — no letters please — to: March Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 28 March 1991. Good luck!

Winner, December 1990

I'm afraid we made a small error when we set this problem — we omitted to restrict the digit multiplier to values greater than unity. In consequence, any 6-digit palindrome with a multiplier of 1 would have been valid. But although many of you pointed this out to us, all the 75 entries received adhered to the spirit of the problem. The two required solutions were therefore:

109989 × 9 = 989901
219978 × 4 = 879912

and this month's winner was Mr A Richardson of Croxley Green in Hertfordshire. Congratulations, Mr R, your prize will be with you shortly if not sooner. Our usual condolences to the also-rans. Don't give up — it could be your turn next.