NUMBERS COUNT

Some sequences generated using the Maximum Prime Factor function, presented by Mike Mudge.

This area of research has been suggested by a regular *PCW* reader, Bruce Halsey of Great Yarmouth, whose preliminary investigation is included below. There appears to be considerable scope for the generation of completely new integer sequences although the theoretical explanation for their behaviour may be difficult to discover.

It is known that a given positive integer, n, has a unique representation as a product of its prime factors; thus, $n = p_1^{m_1} p_2^{m_2} p_3^{m_3} \ldots\ldots p_r^{m_r}$ where m_r is the multiplicity of the prime factor p_r.

Note An efficient algorithm for obtaining this representation for a given n (not necessarily of multiple precision length) is desirable for this investigation.

The 'maximum prime factor' function is defined as: $mf(n) = \max(p_1, p_2, p_3, \ldots\ldots p_r)$, for example:

(i) $306761364 = 2^2 \times 3^3 \times 7^5 \times 13^2$ thus $mf(306761364) = 13$,

(ii) $1983009962 = 2 \times 101 \times 401 \times 24481$ thus $mf(1983009962) = 24481$.

The Halsey Sequence having prime seed p_0

This is defined for a given prime seed, p_0, by the recurrence relation A).....$p_{k+1} = mf(p_k^2+1)$; $k = 0,1,2,3\ldots\ldots$

Thus if $p_0 = 2$, $p_1 = mf(2^2+1) = mf(5) = 5$, $p_2 = mf(5^2+1) = mf(26) = 13$, $p_3 = mf(13^2+1) = mf(170) = 17$, the sequence continues as follows:

29, 421, 401, 53, 281, 3037, 70949, 1713329,.....

Bruce Halsey has used an Atari ST with Fast Basic and its assembler to investigate all p_0 less than 65535. He has found only *one* example of cyclical behaviour, *viz* $p_0 = 89$, $p_1 = 233$, $p_2 = 89$, etc, and thus poses the following questions:

1) Are there any other cycles?

2) Do all prime p_0 lead ultimately to the same sequence?

3) Are any Halsey Sequences unbounded? . . . The above example of p_0 equals 2 might suggest that they are; however, intuitively larger terms seem more likely to have more (smaller) prime factors.

Generalised Halsey Sequences may be defined in an obvious way, each with prime seed p_0.

B).....$p_{k+1} = mf(p_k^2-1)$,

C).....$p_{k+1} = mf(p_k^3 \pm 1)$ etc

D).....$p_{k+1} = mf(2^{p_k} \pm 1)$ etc

E).....$p_{k+1} = mf(3^{p_k} \pm 1)$ etc,

where $k = 0,1,2,3, \ldots$

For each of these sequences, questions regarding cyclical and unbounded behaviour can be asked.

Attempts to generate Halsey and Generalised Halsey Sequences and hence, or otherwise, to describe and explain their behaviour may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 March 1990. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained; together with suggestions for further work on the problem, all in a form suitable for publication in *PCW*. Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, July 1989, Factorial Functions

A detailed study of some of the problems by Reg Bond of 75 Laburnum Crescent, Allestree, Derby DE3 2GS, earns this month's prize. Results obtained include:

Problem 2 Using a combination of modular arithmetic and computing in Fortran Integer*4 arithmetic, there are no solutions for n between 7 and 20,000. Those solutions for n = 3, 4, 5 & 6 are well known.

Problem 5 Reg first considered the modified problem: given m, how small can we make d? He established the theoretical answer that by increasing n and hence r, d can be made as small as we please. A 10-digit display pocket calculator yielded results including:

m = 5:n	**8**	**10**	**12**	**14**
d	**1.99**	**−0.776**	**0.212**	**0.025**

Other submissions deserving mention are: John McCarthy who used QL Superbasic on problem 1 and simultaneously his 'very under-used' Psion Organiser in 12s.f. arithmetic on problem 2. Computing speed proved a severe handicap in this investigation. Gareth Suggett who examined problem 1 for m,n less than 12; problem 2 for n less than 32; referred problem 3 to *PCW* September 1984; problem 4 results included the quadruple products for 8! & 9! and, finally, in problem 5 as Reg Bond, Gareth found the error in Croft's result for n = 20 where d = +0.126.

Review, Lucas Sequences, February 1989, re-opened August 1989

A difficult investigation to judge. The winner, by a very short head, is JR Wordsworth of 30 Branson Crescent, Melton Mowbray, Leicestershire LE13 1ER. The results were obtained on an Atari 520ST (½ megabyte + one single-sided DD) together with Devpack-Atari assembler package (Hisoft), Personal Pascal-compiler by OSS and certain public domain software including Forth 83 and C.

The investigation established Fibonacci primes up to U(1410) in about three days; Williams HC is still comfortably in the lead with U(2971) although this was hinted at by the other 'front-runner', Frank Webster.

The Lucas prime search showed that V(803) has prime factors 199, 151549 and 11899937029 and is *not* prime as stated by Paulo Ribenboim, *The Book of Prime Number Records*, page 287. This should undoubtedly read V(863) with V(1097), a possible candidate to join the other large Lucas primes V(503), V(613) and V(617).

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles. All letters will be answered in due course.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

A Happy and Prosperous New Year to all. Maybe 1990 will be the year in which *you* win the Prize Puzzle!

Winner, October Prize Puzzle

Many of the 200-odd entries received complained that the problem was too easy. As some of you pointed out, the problem gave 7 equations to find only 5 unknowns. Still, it makes a change to have an easy one.

The missing symbol was the ampersand (&) which had a value of 10. The winning card, drawn from the heap, came from James Grinter of Grantham, Lincs. Congratulations, James, your prize is on its way.

This month's quickie

What is the closest relation that your mother's brother's brother-in-law could be to you?

Prize Puzzle

To start the year off, here's a problem in logic which should get the brains and perhaps the micros whirring.

The Foolem Insurance Company held its annual conference on the first day of March, this year. Each staff member had to bring one, and only one, partner.

When the conference opened at 2pm the only Board member to have shown up was the Finance Director with his wife. At that point, those present (there were less than 100) were divided into groups with five people in each.

By 3pm two more people had arrived — the Production Director and his wife. All those present were now subdivided into working parties with exactly four people in each.

By 4pm another two people had come — the Sales Director and his wife. People were now split into groups with three people in each.

By 5pm a further two had arrived — the Managing Director and his wife.

Because of a business meeting, one of the directors had intended that his wife should go to the conference without him and then he would join her an hour later. Fortunately, his business meeting ended early and the arrangement became unnecessary. However, had he done so, it would have been impossible for the people present to subdivide into smaller-sized groups. Which Director was it?

Answers on a postcard or the back of a sealed envelope (no letters, please) and send it to: January Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 31 January 1990. ■

NUMBERS COUNT

A route optimisation problem with a difference, presented by Mike Mudge.

This area of research has been suggested by Jim Gibbons of Thornton Heath, Surrey and is absolutely unique in the seven-year history of 'Numbers Count'. There is not a single algebraic or mathematical symbol in the description of the problem, nor is there believed to be any published material (either empirical or theoretical) relating to its solution. Nonetheless, with a slight stretch of the imagination it can be regarded as a practical problem from the real world of engineering design.

Route optimisation usually involves finding, in some sense, the shortest path from one point to another using an existing network of roads, pipes, cables, and so on. An alternative and well explored problem area is that of connecting in some 'best possible' way a given finite set of stations, vertices, nodes, and so on.

Jim proposes the following: A new town is being designed on a square of unit side. The planners wish to first build a system of roads (negligible in area compared to the available land) all connected but not necessarily to the perimeter. It is required that *no* point of the interior of the square shall be more than half a unit away from a road and that further, for reasons of economy, the shortest total length of road is to be built. What configuration of road should be built?

Some possible ideas:

(a) Replace the square by a finite mesh of points.

(b) Restrict the roads to straight sections.

(c) Define a system of roads empirically using common sense and symmetry.

(d) Test the distance criterion and evaluate the total length with a computer algorithm.

(e) Iterate towards an optimum in some way?

Jim suggests the following possible generalisations:

(I) What happens if the minimum distance is reduced to one quarter of a unit, or indeed any other fraction? Is there a describable generalisation? Prove that any optimal solution must be symmetrical.

(II) Suppose the area of land available was circular and of unit radius (area), or indeed hexagonal as in many well known tiling problems. How would the solution be obtained?

Attempts to generate solutions to Jim Gibbons' problem may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141 to arrive by 1 April 1990. Any communications received will be judged using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained; together with suggestions for further work on the problem, all in a form suitable for publication in *PCW*. Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, August 1989: The Maltese Factor

This problem proved to be by far the most popular yet in both the multiplicity and variety of responses generated, so selecting the prizewinner was consequently a difficult task.

Many correspondents proved the conjecture of Albert Debono in Problem 1 and produced the appropriate computer program. A typical example was JC Hawdon of London, who continued in MacModula-2 on a Macintosh Plus to find the smallest integer expressible in n ways up to n = 47, yielding 45045 = $3^2 \times 5 \times 7 \times 11 \times 13$ in something over two hours.

In complete contrast Dr P McMullen of University College, London required less than two sides of typed A4 to provide complete theoretical analysis of both Problems 1 and 2. He believes that 'Problem 3 may be even deeper than the as yet unsolved Goldbach conjecture, that every even number greater than 2 is the sum of two prime numbers.' Interested readers are referred to 'Numbers Count', *PCW* September 1989.

Colin Singleton used Acorn Basic V on an Acorn Archimedes to display the smallest number which can be expressed as the sum of a series of consecutive positive integers in a specified number of ways, with its factors, up to 575 ways. However, he was able to avoid the problem using consecutive primes!

All entries received have been examined and a sample number forwarded to Albert Debono in Malta. Letters sent to him care of 'MRM at *PCW* will be forwarded immediately.

The final word on The Maltese Factor this month must remain with the worthy prizewinner: Peter Cameron of 70 Godstow Road, Wolvercote, Oxford OX2 8NY. In what he modestly describes as 'a sort of submission to your problem' Peter uses Turbo Pascal V4.0 run on a Zenith Z170 portable.

The extensive computation culminated with the conjecture that the average number of representations as a sum of consecutive primes tends to a constant, the icing on the cake being a theorem, with proof:

The average number of representations of the first x natural numbers as sums of consecutive primes tends to the limit $\log_e 2$ as x tends to infinity.

Well done, Peter!

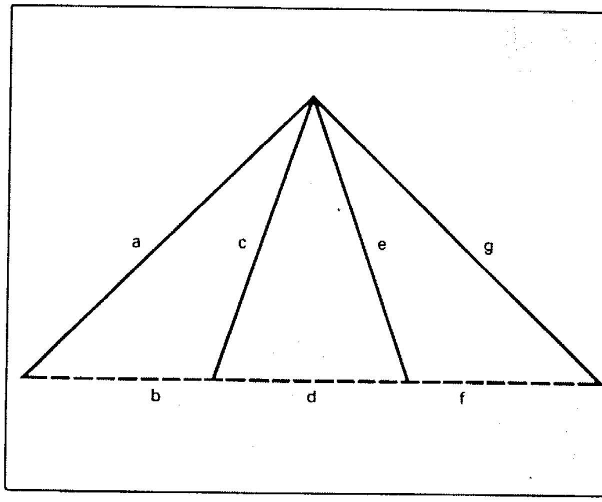
Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles. All letters will be answered in due course.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

This Month's Quickie

If you could fold a sheet of rice paper one thousandth of an inch thick exactly 50 times, how thick would the resulting wad be? Try doing it on your calculator — you'll be surprised at the answer.



Prize Puzzle

A puzzle to start the year off. It was sent to us by Mr Anthony Isaacs of London, and we like it very much. Thanks, Mr Isaacs — we just hope your answer is correct!

In the diagram above (which is not to scale) sides b,d, and f form a straight line, and each side has a length which is a prime number less than 500 — with no two sides being the same length. What is the sum of all the seven sides, given that it is greater than 1000?

Answers on postcards of backs or sealed envelopes — no letters please. Send to: February Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 28 February 1990.

Winner of November 1989 Prize Puzzle

Exactly 200 entries were received, but not all had the correct solutions which were:

Maximum Score: 89

Minimum Score: 52

The problem lent itself very conveniently to computer solution because of the relatively few permutations possible.

The winning card, drawn at random from the pile, came from T Patterson of Leatherhead, Surrey. Congratulations, your prize is on its way. To the remainder of the entrants, keep trying. It could be your turn next. ■

Keeping up with the neighbours

Mike Mudge poses problems generated by the quaintly named Mathematics of Neighbourliness.

This area of investigation was first suggested by Martin Gardner in *Scientific American* some 15 years ago. However, in its present form I am indebted to Michael Meierruth of Milan who describes it as 'representing some good, clean programming fun involving some cute data structures and search algorithms.'

The concept is simple: given an N x N square grid it is required to place 2N pieces onto it in such a way that no three of them lie on a straight line of any orientation — that is, upwards, sideways or diagonally.

The first stage in the investigation is to find *a solution* for N as large as possible. It is at this stage that Michael Meierruth issues a challenge to all 'Numbers Count' readers: he, together with two other persons, responded to Martin Gardner by finding a solution for N=16, but he has recently dug out his code again. Following a four-day run on a Compaq Deskpro 386/20e a solution for N = 17 was forthcoming; the challenge is simply find a solution for N greater than 17.

An obvious extension of this work is to analyse the sequence of integers, counting the number of distinct solutions (not counting rotations and mirror images) as a function of N. Is this a finite sequence? That is, is there a maximum N beyond which no solutions exist?

Returning briefly to the title, the only obvious practical application is that if houses are arranged in this way on a square grid, then from each house some piece of every other house on the estate can be seen. This surely is neighbourliness!

A conceptually difficult extension which has occurred to the writer is that of pieces located within an M-dimensional hyper-cube. However, it is clear that a detailed investigation of the proposed 2-dimensional problem is essential before this generalisation is even considered.

Attempts to generate solutions for any values of N (and M) may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 May 1990. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, Goldbach's Conjecture

Gareth Suggett defines G(n), n even, as the number of Goldbach decompositions of n and computes this in two distinct ways.

(1) Useful only for small n; given a table of primes add these in pairs and count the number of 'hits' on each value of n. Print out the total number of hits once all the pairs of primes being considered have sums greater than n.

(2) Partition n = A + B and then check A and B for primality. This was run up to G(1000) = 32.

A combination of these algorithms took Gareth to $G(10^6)$ = 5402 at which point he found G(n) approaching 0.6 of the Hardy and Littlewood asymptotic formula for G(n) (see either Guy, *Unsolved Problems in Number Theory*, p58, or Ribenboim, *The Book of Prime Number Records*, p321). Can any reader either confirm or fault this result? It appears to be particularly interesting.

Gareth then modified his programs to consider only primes belonging to prime pairs and found the interesting fact that those numbers with no representations using this restricted set of primes appear to cluster in groups thus: 0,2,4;94,96,98;400,402,404; 514,516,518;784,786,788; . . .

However, using the usual philosophy of 'suitable subjective criteria', the prizewinner this month is WJ (Bill) Smith of Plymyard, The Barrow, Boddington, Cheltenham GL51 0TL. Bill first of all wishes to draw attention to the **non-profit making** operation known as I-APL Ltd of 2 Blenheim Road, St Albans, Herts AL1 4NR, whose stated aim is to spread awareness of APL as a problem solving tool. I-APL appears to be aimed at the educational market and is ported across Spectrums, BBCs and so on as well as PCs; it has been designed to exist within a 64k workspace and is therefore written for compactness rather than speed of operation.

Bill used I-APL on an Opus PC III with its 8088 running at 10MHz and went some way down the line with each problem posed, being constrained by I-APL's limited workspace on each investigation.

The language is built around array processing of 32k so does not provide a lot of space once the arrays get big. How might this be overcome?

Any replies direct to Bill please (or to the 'Letters' column of *PCW*) who will willingly supply copies of his programs to any interested readers.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles. All letters will be answered in due course.

Brainteasers courtesy of JJ Clessa.

Winner, December 1989

The Xmas number crossword proved to be quite popular. Well over 200 entries were received, mostly correct. But we could only have one winner, and that lucky person, drawn from the pile, was Mr IJ Inglis of High Wycombe, Bucks. Our congratulations to you, Mr Inglis, your prize is on its way. To everyone else, keep trying — maybe this month is your turn to win. The solution is given below.

Quickie

Our thanks to Miss Nicola Sparnon of Casterton School, Cumbria, for this month's puzzle.

Can you, by a single stroke of the pen (or pencil) make the following equation correct?

5 + 5 + 5 = 550

Prize Puzzle

This should burn up the micros:

16 contains 2 digits and it is also a perfect square.

125 contains 3 digits and it is also a perfect cube.

4096 contains 4 digits and it is also an exact 4th power.

59049 contains 5 digits and it is also an exact 5th power.

What is the largest number which has this property?

Answers on postcards or backs of sealed envelopes (no letters please) to: March Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 28 March 1990.

[1] 2	■	■	[2] 2	[3] 6	0	[4] 1	■	[5] 5	[6] 1	■	[7] 8
[8] 6	[9] 7	[10] 6	■	2	■	0	■	[11] 3	6	■	0
■	[12] 8	0	[13] 1	■	[14] 2	4	0	3	■	[15] 2	1
[16] 1	■	[17] 8	0	1	■	0	■	[18] 6	9	3	0
[19] 3	0	4	2	■	■	[20] 4	[21] 4	1	■	1	■
2	■	■	■	[22] 1	0	■	0	■	[23] 1	0	[24] 5
[25] 6	[26] 9	3	■	6	■	[27] 2	1	■	■	■	6
■	3	■	[28] 6	3	[29] 2	■	■	[30] 5	[31] 7	0	0
[32] 3	3	6	7	■	6	■	[33] 6	2	9	■	4
[34] 7	6	■	[35] 6	3	5	6	■	[36] 9	3	[37] 2	■
2	■	[38] 1	2	■	6	■	[39] 6	■	[40] 2	3	[41] 1
1	■	[42] 9	6	■	[43] 9	2	1	6	■	■	5

NUMBERS COUNT

Pseudoprimes, Carmichael Numbers and an Ancient Chinese Fairy Tale, from Mike Mudge.

Revision note on congruence and modulus notation
Given three integers (whole numbers), A, B & C, we write $A \equiv B(C)$ (read A is congruent to B modulus C) if and only if A and B leave the same remainder when divided by C. It clearly follows that A — B is a multiple of C, *viz* $A - B = kC$ for some integer k, either positive or negative. For example $273 \equiv 84(9)$ since $273 = 30\times9 + \underline{3}$, $84 = 9\times9 + \underline{3}$ and clearly $273 - 84 = 189 = 21\times9$.

The ancient Chinese are alleged to have considered possible solutions of $2^n \equiv 2(n)$ and further to have suspected that such n were prime. This 'fairy tale' is to be found in a paper by JH Jeans, in the *Messenger of Mathematics*, 27, 1897/8, dating the study of this congruence from the time of Confucius.

However, in his recent *Book of Prime Number Records*, Springer-Verlag 1988, Paulo Ribenboim destroys this myth firstly by observing that the Chinese mathematicians of the time had never formulated the concept of a prime number, and secondly by relating his own investigations of the literature revealing the source of an 'erroneous translation'.

Definition n is defined to be a **pseudoprime** to **base a** if: $a^{n-1} \equiv 1(n)$, and further n is not prime. Hence Ribenboim refers to the congruence $2^n \equiv 2\ (n)$ as the 'pseudo-Chinese congruence on pseudoprimes'!

In 1819 Pierre Sarrus found $341 = 11\times31$ as the smallest pseudoprime base 2. Pseudoprimes, to a given base, are quite rare. There are 882,206,716 primes less than 2×10^{10} but only 19,865 **pseudoprimes, base 2**, in the same range.

The sequence of pseudoprimes, base 2, commences: 341, 561, 645, 1005 . . . There are indeed even numbers satisfying $2^n \equiv 2(n)$, called **even pseudoprimes.** However, it should be noted that they are relatively rare, the smallest being $161038 = 2\times73\times1103$ due to Lehmer 1950, the next being 215326. There are an infinity of even pseudoprimes, each must have at least two odd prime factors (Beeger 1951).

It is not known if there are infinitely many square pseudoprimes, the smallest examples are $1194649 = 1093^2$ and $12327121 = 3511^2$.

PROJECT A Determine the smallest pseudoprime to each of the bases 2, 3, 5 and 7. Hence, or otherwise, find the smallest pseudoprime for the simultaneous bases 2,3; 2,5; . . . then for 2,3,5; 2,3,7; . . . and finally for the quadruplet of bases 2,3,5,7.

Definition n is a Carmichael Number if:
$a^{n-1} \equiv 1(n)$ for *every* integer **a** satisfying $1 < a < n$ and such that a is relatively prime to n: that is, a and n have no common factors (divisors) other than unity, and further n is not prime.

These numbers were first investigated by RD Carmichael in 1912 — he called them absolute pseudoprimes. (It should be noted that pseudoprimes are sometimes called Poulet Numbers after the investigations of P Poulet in 1926 and later in 1938).

It is not known if there exist infinitely many Carmichael Numbers. The sequence of such numbers commences: 561, 1105, 1729, 2465 . . .? The largest known Carmichael Number is believed to be that due to R Dubner in 1985 wit 1057 digits while in 1978 M Yorinaga found eight Carmichael Numbers each with 13 prime factors.

PROJECT B Construct and implement a computer algorithm to generate all Carmichael Numbers less than 3×10^5, say. Further consider the problem of verifying that a given number is indeed a Carmichael Number and hence verify the result of J Chernick, 1939, that if $m\geq1$ and the three factors $6m + 1$, $12m + 1$ and $18m + 1$ are all prime then their product is indeed a Carmichael Number.

Can this result be generalised to construct Carmichael Numbers having p prime factors?

Attempts at either or both of the above projects may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel (09746) 548, to arrive by 1 June 1990. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the st' contribution arriving by the closing date.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs and run times, and a summary of the results obtained; together with suggestions for further work in this area, all in a form suitable for publication in *PCW*. Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, The Khinchin Constant
Khinchin's Constant, 2.6854520010653064 . . . is evaluated to some 68 decimal places in *MTAC*, vol 14, p371, 1960. Its continued fraction expansion, commencing 2;1,2,5,1,1,2,1,1,3,10,2 . . . is obtained in *MTAC*, vol 20, p446, 1966; while both quantities are available from Sloane NJ, *A Handbook of Integer Sequences*, Academic Press 1973, as entries 609 p73 and 47 p36 respectively.

The slow convergence, both of the infinite product and of the associated infinite series, proved to be a stumbling block for a number of contributors. Gareth Suggett using a BBC Micro and 323628 terms obtained K as approximately 2.6854(6) while Frank Webster using an Electron programmed in BBC Basic took 10^5 terms in 94 minutes and 10^6 terms in 14 hours to find K to 8 decimal places and its continued fraction to 13 partial quotients. A quasi-empirical argument by Frank led to 2.2 as a crude approximation to the limit described in Problem (3).

However, the very worthy prizewinner this month is Mathias Meuser of Annette, Kolb, Anger 15, 8000 Munich, West Germany for a combination of theoretical and computational investigations. The latter used interpretive Basic on a Tandy Model 100 with 24k of memory: obtained 10 decimal places eventually but three correct with 100 terms and two infinite integrals!

Readers interested in continued fractions are recommended to read A Ya Khinchin, *On Continued Fractions*, Dover paperback, together with the *MTAC* references given above.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles.

Mike Mudge investigates Harshad Numbers and non-zero Harshad Numbers.

This topic first appeared in 'Numbers Count' in August 1983 on which occasion the associated prize award was withheld. Come along, *PCW* readers: there is no essential mathematical background other than basic arithmetic, but there is considerable scope for extending the current knowledge of this subject area.

A **Harshad Number** is a positive integer which is a multiple of the sum of its digits. For example, 4975 is a H-number because $4975 = (4+9+7+5) \times 199$, similarly 88998 is a H-number because $88998 = (8+8+9+9+8) \times 2119$.

A **Non-Zero Harshad Number** is a H-number, none of whose digits is zero. Thus both of the H-numbers shown above are NZH-numbers whereas 40975, although a H-number, is not an NZH-number on account of the zero digit which it contains.

The properties required for H-numbers and NZH-numbers are clearly **base-dependent** which makes them particularly interesting to the empirical number theorist using a personal computer. For example, 190 is a H-number since $190 = (1+9+0) \times 19$: it is not a NZH-number. However, if we move from base 10 to base 3 (in which, incidentally, the first generation of Russian computers worked, rather than base 2 or binary) we find $190_{10} = 21001_3$: that is, $190 = 2 \times 3^4 + 1 \times 3^3 + 1 \times 3^0$. The sum of the digits is now 4 and since 190 is *not* an integer multiple of 4 the property of being a H-number has disappeared. It returns again in base 4 where $190_{10} = 2332_4$, the digit sum again being 10. However, 2332_4 has no zero digits and is thus an NZH-number, unlike 190_{10}.

Project A Design and implement an algorithm for computing the smallest H-number, base 10, having digit sum d for d = 10,11... Note that digit sum 1,..9 is trivial yielding smallest H-numbers 1,..9 respectively (see below).

Questions of immediate interest include:
(i) What are the smallest H-numbers with digit sums 40 and 41?
(ii) Is 2918999999999 the smallest H-number with digit sum 101?
(iii) Is 8587 followed by 27 nines the smallest H-number with digit sum 271?

Project B Extend the algorithm of A above to find the smallest NZH-number when this is different from the smallest H-number. Careful!

Project C Extend the results of A and B above to any number base,b, to be input together with the required digit sum,d.

Conjecture to be considered: 'The number of NZH-numbers for a given d is finite' . . . If this is true, is it realistic to list all such numbers.?

Thought for the month

Several years ago Peter Stanbury, a numerologist from Tunbridge Wells, wrote to me with the following observation: 'If a,b,c, and d are all consecutive primes larger than 5 then the following formula produces a surprising number of primes: $ab - (c + d)$. For the first 18 values of 'a' the formula produces 15 primes, this ratio decreasing to 23 out of 31, 35 out of 67, and 48 out of 119. I have not checked it further, but you might wish to do so.'

Do readers agree with these numbers, wish to extend them, and is this a surprising number of primes?

Attempts at the above projects together with observations on the thought for the month may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel: (09746) 548 to arrive by 1 July 1990. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained; together with suggestions for further work in this area, all in a form suitable for publication in *PCW*.

Test Data

Digit sum	10	14	18	22	26	42
Smallest H-number	190	266	198	2398	1898	88998

Review, Greedy Sequences

A Parry of Porth used BBC Basic on an Archimedes A310 to find the first 258 terms of the Sidon Series less than 350000. Gareth Suggett did his usual comprehensive attack on all the problems but the total number of substantive responses was disappointing.

The prizewinner, however, is a quartet of Portugese students referred to as Luis Lampreia *et al* of Rua Jacinto Nunes 5-4 Esq. 1100 Lisboa, Portugal, using a Philips MSX-2 together with assembler on a Spectrum. Details of the group's work are available from the proposer, Peter Cameron, of 70 Godstow Road, Wolvercote OX2 8NY.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future articles.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

This Month's Quickie

No answers, no prizes. Can you find which of the following numbers is the odd man out?
968 583 375 286 781 605 209 946

Prize Puzzle

And now for a crossnumber puzzle with a difference. Our thanks to Peter Stanton of London for this novel idea.

Each answer to the 26 clues is a number in which exactly two identical digits are present — no more and no less. The clue itself shows the sum of the digits in the answer.

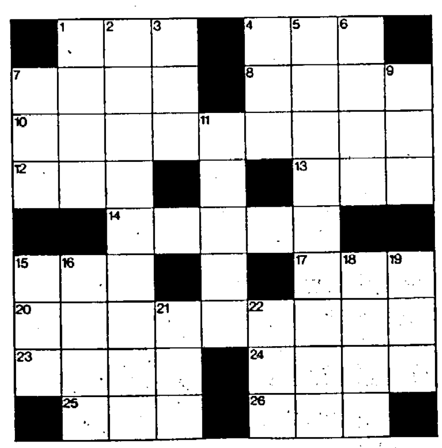
When you have completed the puzzle paste the completed grid to a postcard or the back of a sealed envelope — please don't send letters — and post it to: May Prize Puzzle, PCW Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 30 May 1990.

Winner, February 1990

Where were all those entrants who invariably cry 'Too easy!'? There were less than 40 entries for the February problem. One reader gave up '. . . after 10 hours coding and 4 hrs 45 mins running on an Amstrad PC1640. . .'. Another wanted to see a program for solving it since he estimated that it would need six months on his micro to do it. I'm sorry Mr N, but everyone obeyed the rules this time and no letters were sent (usually we get at least six or seven letters which invariably contain program listings), but since the puzzle was submitted by Mr Anthony Isaacs, perhaps he might wish to offer one.

The answer to the problem, found after much number crunching, was 2101 with sides of 449, 151, 457, 17, 461, 79 and 487. The winning card came from Scotland — from Mr P Bradbeer of Inverkeithing, Fife. Congratulations, Mr Bradbeer, your prize will be sent forthwith.

Clues Across
1a 12
4a 17
7a 33
8a 7
10a 40
12a 8
13a 10
14a 35
15a 15
17a 21
20a 51
23a 12
24a 13
25a 21
26a 13



Clues Down
1d 16
2d 53
3d 12
4d 20
5d 37
6d 13
7d 9
9d 19
11d 21
15d 26
16d 22
18d 15
19d 18
21d 5
22d 6

Mike Mudge investigates the relationships between Quadratic Partitions and Cubic Residues.

This research area has been suggested by NV Meeres of Esher, who explored it in considerable detail in 1982. Although certain pure mathematical concepts are required these are carefully defined, with simple numerical examples, in Appendix A.

The objects of interest are those prime numbers, p, congruent to unity modulo 6. That is, 7, 13, 19, 31, 37, 43, 61, 67, 73. . . The overall task, *to be considered in several stages*, is to attempt to use quadratic partitions of p, and related integers, to generate the complete set of cubic residues of p.

Notice that each p has a unique quadratic partition of the form: $3\times A^2 + B^2$. For example, $7 = 3\times 1^2 + 2^2$, $13 = 3+2^2 + 1^2$, $19 = 3\times 1^2 + 4^2$ . . . Each p has (p − 1)/6 pairs of cubic residues; $(R_1, p-R_1)$, $(R_2, p-R_2)$ . . . while the remaining integers less than p−1 fall equally into two sets of pairs depending upon whether their indices are congruent to +1 or −1 modulo 3. For example, the cubic residues of 19 are (1,18), (7,12), (8,11). The remaining integers less than 18 are (2,17), (3,16), (5,14) with index congruent to +1 mod 3 and (4,15), (6,13) (9,10) with index congruent to −1 mod 3.

In attempting to identify the sequence $R_1, R_2, \ldots$ and hence to generate the complete set of cubic residues of p, many famous mathematicians were led to study the quadratic partition $4p = 27C^2 + D^2$, for which C×D and its factors are always cubic residues of p. For example, p=19, $4p=76=27\times 1^2+7^2$; C×D=1×7. p=307, $4p=1228=27\times 6^2 + 16^2$; C×D=96. The cubic residues of 307 include all of the factors of 96, and indeed of 96^2 (reduced modulo 307 when appropriate).

Now, if two primes are both cubic residues of p their product is also, but a composite, qr, may also be a cubic residue when neither of its factors is. For example, 10 is a cubic residue of 37 although 2 & 5 are not.

The case when qr = 10 leads to the conjecture that 10 is a cubic residue of p if and only if A×B (recall $p=3\times A^2+B^2$) is divisible by 10. For example, (p,A,B): (73,4,5), (79,5,2), (103,1,10), (349,10,7) . . .

Two further observations of NV Meeres are:

(a) Any integer congruent to ±1 mod 9 is a cubic residue (CR) of p if (though not only if) it is a factor of A×B. For example (p,A,B:CR's): (67,1,8;1,8), (97,4,7;28), (181,2,13;26) . . .

(b) The least multiples of A and B congruent to ±1 mod 9 are cubic residues of p. For example, (p,A,B:CR's): (61,2,7;8,28), (271,5,14;10,28), (421,10,11;10,44) . . .

Enough theory for this month!

Project A Design and implement a computer program to obtain the primes, p, congruent to unity modulo 6 together with the associated quadratic partitions $p = 3\times A^2 + B^2$ and $4\times p = 27\times C^2 + D^2$.

Project B Design and implement a computer program to verify that C×D and its factors are indeed CR's of p.

Project C Design and implement a computer program to test both the conjecture regarding the existence of a CR of 10 and A×B (the late Professor Goodstein of Leicester University verified this for p less than 10000 in 1964), also the observations (a) & (b) due to NV Meeres above.

Attempts at some, or all, of the above projects may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel (09746) 548 to arrive by 1 August 1990.

APPENDIX A Some pure mathematical concepts and definitions.

(i) Modulus & Congruence Two integers m & n are said to be congruent modulo a third integer r if and only if they differ by a multiple of r. That is, m & n leave the same remainder on division by r. We write m = n(r), for example, 17=65(4), because 17−65 = −48 = −12×4.

(ii) Quadratic Partition A quadratic partition of a given integer,K, is simply an expression of two squares. For example, $68 = 3\times 4^2 + 5\times 2^2$ is a quadratic partition of 68.

(iii) Cubic Residue (CR) If p greater than 2 does not divide a and there exists an integer n such that $a \equiv n^3(p)$ then a is a cubic residue of p. For example, if p = 19 then $11 \equiv 5^3(19)$ and 11 is a CR of p. Notice that 11 − 125 = −114 = −6×19.

(iv) Order The order of a modulo p is the smallest power of a which is congruent to 1 modulo p.

Review, December 1989, 'Some famous integer sequences'

This problem area proved to be popular. Among the submissions worthy of special mention are the following: Jim Waterton who, although restricted by the single precision arithmetic of his QL, did an in-depth study; Gareth Suggett, who computed up to R_{25}, B_{26}, E_{25} and S(13,13) with very substantial factorisation; Luis Lampreia and three fellow Portugese students who generated a detailed submission using Basic on a Philips MSX−2; and Frank Webster's achievements include S(100,4), 158 digits in 11 seconds, and R_{100}, also 158 digits in 2.4 seconds using both Basic and Assembler on an Acord Electron.

Readers are also encouraged to read 'Micromaths: some discoveries about tan x' by Adrian Oldknow, *Teaching Mathematics and its Applications*, vol 8, no 3, 1989, pp135-141.

The very worthy prizewinner this month is WE Thomson of Woodhaven, Leiston Road, Aldeburgh, Suffolk IP15 5PX. This submission 'turned out to be more of an exercise in algebra than in number-crunching'. Mr Thomson 'first came upon subfactorials in a practical problem about 50 years ago'. Space does not permit details of the balance between algebra and computing, but these are available on request.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

This Month's Quickie

No answers, no prizes, and no tricks this month. Using each of the digits 1-6, make a 6-digit number whose:

1. First and last digits are odd.
2. Second digit is twice the fourth.
3. 5th digit is twice the 2nd.
4. 3rd digit is one more than the last.

Prize Puzzle

This shouldn't be too difficult. Even if you can't do it, you've still got a one in six chance of being correct if you make a guess!

Albert, Barry and Charlie are married to Alice, Betty and Celia, but not necessarily respectively. One day the three couples went to the Post Office to buy stamps. Each man spent 63p more than his wife. Each person bought as many stamps as they paid in pence per stamp. Albert bought 23 more stamps than Betty and Barry bought 11 more stamps than Alice.

Who is married to who?

When you have completed the puzzle, write the solution on a postcard or the back of a sealed envelope — no letters please — and post it to: June Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 30 June 1990.

Winner, March 1990

A good response to our number problem — about 150 entries, practically all of them containing the correct solution which was 9^{21} or, if you prefer to be longwinded: 109,418,989,131,512,359,209

Despite the relatively high number of entries, we pulled out our first ever triple winner, Mr G Langley of Canterbury who, if our records are correct, was a winner in December 1985 as well as November 1986. We assure you that he is neither a relative, nor anyone we know!

Congratulations, Mr Langley, another prize is on its way to you. Perhaps the Post Office will soon be able to offer us a quantity discount for prizes sen' to your address. Meanwhile, our condolences to all the others who lost this time, and do keep trying.

Mike Mudge poses some problems in amicable and quasi-amicable numbers.

Given any positive integer, n, we define s(n) to be the sum of all of the proper* divisors of n (*that is, excluding n itself), hence $s(6) = 1+2+3 = 6$ while $s(28) = 1+2+4+7+14 = 28$.

Readers may recall that n is defined to be a 'perfect' number if and only if (iff) $s(n) = n$, ref *PCW* October 1983. Thus from the above examples both 6 and 28 are perfect numbers.

Extensions of this definition include the following:

1) n is **quasi-perfect** or **almost perfect** according to $s(n) = n+1$ or $n-1$ respectively.

2) n is **2–hyper-perfect** iff $n = 2 \times s(n) - 1$.
n is **3–hyper-perfect** iff $n = 3 \times s(n) - 2$. ....
n is **r–hyper-perfect** iff $n = r \times s(n) - (r-1)$.

3) n is **multiply perfect** with **index of perfection** k iff $s(n) + n = k \times n$. For example, 120 is multiply perfect with index of perfection 3 while 30240 is multiply perfect with index of perfection 4.

4) $n_1, n_2, n_3, \ldots\ldots n_t$ are **sociable** numbers with **index of sociability** t, iff $s(n_1) = n_2$, $s(n_2) = n_3, \ldots, s(n_t) = n_1$. For example, 14316 is a sociable number with index of sociability 28.

(*Note*. Perfect numbers are simply sociable numbers with index of sociability 1.)

The central topic for investigation this month focuses on sociable numbers with index of sociability 2.

Definition The pair of positive integers (m,n) is defined to be an **amicable** pair iff $s(m) = n$ and $s(n) = m$; clearly equivalent to m and n being sociable numbers with index of sociability 2.

The first, that is smallest, amicable pair (220,284) has been known since ancient times. Dickson (1952) proved that there exist only five amicable pairs in which the smaller number is less than 6233, but by 1973, 1107 amicable pairs were known, the largest being a pair of 25-digit numbers.

Subsequently, four further amicable pairs (at least) have been discovered with 32, 40, 81 and 152-digit numbers. The existence of relatively prime amicable pairs, that is where the two integers have no common divisor other than unity, has been proved theoretically — Kanold (1953), Hagis (1969-'70). However, none have been found in a search up to 10^{60}.

Definition The pair of positive integers (m,n) is defined to be a **quasi-amicable** pair iff $s(m) = n+1$ and $s(n) = m+1$. Forty-six quasi-amicable pairs are known, with the smallest member less than 10^7. The sequence begins with (48,75);(140,195) and 'ends' with (8829792,18845855); (9247095,10106504).

Project A Design and implement a computer program to obtain amicable pairs, hence determine as many as possible of the 1107 such pairs referred to above.

Project B Design and implement a computer program to obtain quasi-amicable pairs and attempt to discover the 46 such pairs whose smallest member is less than 10^7. Extend this result if possible.

Project C Design and implement a computer program to tackle the more general problem of sociable numbers with general index, t.

Thought for the month: Things you may not have known about powers of 2. $2^{10} = 1024$ is the smallest power of 2 containing a zero. 2^{53}, which is approximately $9.007199255 \times 10^{15}$, is the smallest power of 2 containing two *consecutive* zeroes. What is the smallest power of 2 containing *nine* consecutive zeroes? The result is known thanks to Mr C Tooth, *PCW* February 1986. What are the corresponding results for consecutive ones, twos,... and powers of 3,4,5....?

Attempts at some, or all, of the above projects together with reactions to the thought for the month, may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel (09746) 548, to arrive by 1 September 1990. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

Review January 1990

Thanks are due to Bruce Halsey of Great Yarmouth for a most interesting area of investigation. Several contributors dealt swiftly with $p_{k+1} = mf(p_k^2 - 1)$ which ultimately cycles at (2,3).

The only complete treatment of $p_{k+1} = mf(p_k^2 + 1)$, the 'fundamental' Halsey sequence, was due to Reg Bond of Derby who established that cycling can *only* occur around two distinct primes and that these are $p_0 = u_r$ and $p_1 = u_{r+2}$ where u_r and u_{r+2} are odd Fibonacci Numbers, r greater than or equal to 11 if $(3, (r+4)/3) = 1$ and r greater than or equal to 14 if $(3, (r+4)/3)$ greater than 1..

The solution for r less than or equal to 1000 are (i) $u_{11} = 89$, $u_{13} = 233$ as quoted by Halsey, (ii) u_{431}, u_{433} with 90 and 91 digits respectively, and (iii) u_{569}, u_{571} with 119 digits each.

Gareth Suggett draws readers' attention to $p_{k+1} = mf(2xp_k + 1)$ where the cycle (5,11,23,47,19,13,3,7) 'appears to attract everything'.

However, the worthy prizewinner is Frank Webster of 125 Coniston Grove, Middlesbrough, Cleveland TS5 7DF, with extensive studies of all problems A)...E) using an Acorn Electron programmed in BBC Basic with some routines in assembly language. Space does not permit a detailed discussion of the results, but they may be obtained from Frank or myself, in summary, on receipt of a suitable sae.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

This Month's Quickie

No answers, no prizes, but slightly more difficult than usual. On the island of Monga-Monga there is no such thing as money so the natives barter with fruit. The going price for three goats and a mule is two cows, and the going price for three cows, two mules, and a goat is 25 sheep. How many sheep will you get for each of the other three animals?

Prize Puzzle

And now for something slightly more difficult that should stretch the micros — or at least, the grey matter.

In a boys' school in Glasgow one of the classes contained exactly 30 boys, 15 of which were Celtic fans, the other 15 supported Rangers. As a punishment for a misdemeanour, for which the culprit could not be found, the teacher decided to detain exactly one half of the class over a Saturday afternoon period so that they would miss the 'derby' match. In order to be fair, he arranged the entire class in a circle and proceeded to count clockwise, with every 13th boy dropping out for the Saturday detention, until 15 boys had been removed. However, a clever Celtic supporter told his friends where to stand so that only the Rangers fans would be punished.

What are the positions (1 to 30) where the Celtic fans should stand at the start of the count so that none of them will miss the game?

Answers on postcards or backs of sealed envelopes — no letters please. Send to: July Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 31 July 1990.

Winner, April 1990

Due to an unfortunate glitch in the system, no puzzle appeared in the April issue. Due to the same glitch we didn't give the winner of the January puzzle, so we'll do that now. (Phew, and we thought our puzzles were complicated!)

Winner, January 1990

There was a mediocre response to our January problem and yet it was quite easy — perhaps that was the trouble. Less than 80 entries were received, about 10 of which were incorrect. The answer to the problem was the Sales Director, and the winning card drawn from the batch came from Nairobi, Kenya — from Mr (?) Agola Gregory. Congratulations, Mr Gregory, your prize will be sent forthwith. To the also-rans — keep trying; maybe this month is your turn.

Part two of quadratic partitions and cubic residues explored by Mike Mudge.

This month readers are invited to continue the exploration of this fascinating research area suggested by N V Meeres of Esher and introduced as Part I, *PCW* June 1990. The pure mathematical concepts required are carefully defined, with simple numerical examples, in Appendix A, reproduced here from Part I for completeness.

Throughout this work, p denotes a prime number congruent to 1 modulo 6, and hence having associated unique quadratic partitions (QPs) of the form $p = 3xA^2 + B^2$ and $4p = 27xC^2 + D^2$ (Part I, Project A).

Now consider the composite qr = 6 and ask when this is a cubic residue (CR) of p. The QP $27xC^2 + D^2$ becomes useful in cases where neither 2 nor 3 is a factor of CxD. It can be shown that under these conditions either C + D or C − D must be congruent to 0 modulo 6. If the quotient on division of C ± D is even, then 6 is found to be a CR of p; otherwise 12 is a CR and 6 is not.

Other values of 2q, such as 22 and 34, behave in the same way, provided that, like 6, they are congruent to ±1 modulo 7 and that C + D or C − D are congruent to 0 modulo 4q. Here the primes to be considered will include some where C is divisible by 3, but 2 and q must not be factors of either C or D.

Project A Investigate the sequence of primes **7, 13, 97, 139, 151, 313, 673, 751, 769, 937, 1063, 1117, 1321,** . . . using the above results to determine possible CRs. Hint: Results should begin **(6),(12),(12),(6,44)** (44 also 3 & 19 by part I), . . .

Now if $2q \equiv \pm 3 \bmod 7$ it is a CR when $C \pm D \equiv 0 \bmod 2q$ but $C \pm D \not\equiv 0 \bmod 4q$. Similarly, if $C \pm D \equiv \bmod 4q$ then 4q is a CR but not 2q.

Project B Investigate the sequence of primes **73, 97, 103, 193, 349, 373, 571, 661, 673, 757, 1123, 1429, 1543,** . . . using the above results to determine possible CRs. Hint: results should begin **(10),(20,12),(10),(20),(38,6)** . . .

Now if $2q \equiv \pm \bmod 7$ it cannot be a CR of this class of primes, for it is found that q itself is a CR whenever $C \pm D \equiv 0 \bmod q$.

Project C Investigate the sequence of primes **163, 709, 877, 991, 1783, 1867,** . . . using the above results to determine possible CRs. Hint: results should begin **(13),(13),(29),(29),(43),(41),** . . . Recall that the object of this investigation is to attempt to generate a complete set of CRs for any given prime number congruent to 1 modulo 6. With this object in mind readers are encouraged to consider, in both theory and practice:

Project D For the case 2q = 14 the key to the analysis is the QP $4p = 3XE^2 + F^2$ where E = A ± B whichever is not congruent to zero modulo 7. How does this key provide information on CRs?

Attempts at some, or all, of the above projects may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel (09746) 548 to arrive by 1 October 1990. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded, by *PCW*, to the 'best' contribution arriving by the closing date.

APPENDIX A Some pure mathematical concepts and definitions.

(i) Modulus & Congruence Two integers m & n are said to be congruent modulo a third integer r if and only if (iff) they differ by a multiple of r: that is, m & n leave the same remainder on division by r. We write **$m \equiv n(r)$**, or **$m \equiv n$ mod.r**, for example **$17 \equiv 65(4)$** because **7-65 = -48 = −12x4.**

(ii) Quadratic Partition (QP) A QP of a given integer, K, is simply an expression of K as the sum of multiples of two squares, for example **$68 = 3x4^2 + 5x2^2$** is a QP of 68.

(iii) Cubic Residue (CR) If p greater than 2 does not divide, a, and there exists an integer n such that **$a \equiv n^3(p)$** then a is a CR of p. For example, if **p = 19** then **$11 \equiv 5^3(19)$** and 11 is a CR of p. Notice that **11 − 125 = −114 = −6x19.**

(iv) Order The order of an integer, a, modulo p is defined as the smallest power of a which is congruent to 1 modulo p.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles.

LEISURE LINES

JJ Clessa's brainteasers.

Prize Puzzle

This is a micro-whirling puzzle which I used once before, many years ago.

Three friends, Alan, Bert and Colin, possess a Mercedes, a Metro, and a Moped respectively. The three are discussing their vehicles' mileages when Alan reports that the 6-figure speedometer on his car is showing a palindromic value of 006600 miles. 'That's interesting,' says Bert, 'mine is also palindromic — 18981 miles, although my speedometer only has 5 figures'. 'Well, I never' says Colin, 'my 4-figure Moped speedo is showing 5335 miles, so we're all palindromic at the same time — I wonder if we're ever likely to get such a coincidence again'.

Well, of course, we could never answer that since each vehicle does a different weekly mileage from the others. But we do have four different questions for you. Supposing that all three speedometers were fitted to Alan's Mercedes and were equally accurate, what is the least number of miles that would elapse before:

1 Alan and Bert's mileages were mutually palindromic again.
2 Bert and Colin's mileages were mutually palindromic again.
3 Alan and Colin's mileages were mutually palindromic again.
4 All three mileages were mutually palindromic again.

Answers on postcards or backs of sealed envelopes — no letters, please — to: August Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 31 August 1990.

Winner, May 1990

A good response to the unusual number crossword in the May puzzle — and most people seemed to enjoy it. Almost all of the 80 entrants submitted the correct solution, so it was up to our random selection process (find the postcard with the fivers attached) to give us our winner, who this month is Mr FC Jessop of Bridport in Dorset. Congratulations Mr Jessop, your prize will be sent to you shortly. Our usual condolences to the also-rans with our usual plea — don't give up.

The correct solution is shown.

■	[1] 5	[2] 5	[3] 2	■	[4] 9	[5] 4	[6] 4	■
[7] 7	9	9	8	■	[8] 2	1	1	[9] 3
[10] 1	1	4	2	[11] 5	9	3	7	8
[12] 1	1	6	■	1	■	[13] 1	1	8
■	■	[14] 7	9	8	9	2	■	■
[15] 9	[16] 3	3	■	1	■	[17] 8	[18] 5	[19] 8
[20] 9	9	8	[21] 1	6	[22] 4	7	2	5
[23] 8	1	2	1	■	[24] 1	5	2	5
■	[25] 9	9	3	■	[26] 1	6	6	■

Mike Mudge poses some problems relating to the primitive roots of (safe) prime numbers.

The area of investigation this month has been suggested by PS Brady of Wrexham, Clwyd who sees it as 'the topic which interests me most at present (and probably for some time to come)'; thus there is obviously much scope for investigative computing here.

Some mathematical prerequisites

A **prime number**, p, is an integer (whole number) which is only exactly divisible by itself and unity (one), for example, 2,3,5,...31,..613... A **safe prime number** is a prime of the form 2p + 1, for example, 7 = 2×3 + 1, 83 = 2×41 + 1, 503 = 2×251 + 1.

Two integers a & b are said to be **coprime** if and only if (iff) they have no common divisor (factor) other than unity. We write (a,b) = 1. (In general (a,b) denotes the highest common factor (HCF) of a&b.) 16 and 57 are coprime while (96,172) = 4.

Two integers a & b are said to be **congruent modulo** a third integer,m, iff their difference is exactly divisible by m; alternatively iff they each leave the same remainder on division by m. We write a≡b(m), for example, 80≡25(11) because 80 − 25 = 55 = 5×11.

If there exists an integer x such that $x^2 \equiv a(m)$ where (a,m) = 1 then a is called a **quadratic residue** modulo m. In this investigation m will be replaced by a prime, possibly a safe prime,p. For example, 13 is quadratic residue modulo 23 because $6^2 \equiv 13(23)$.

The **Legendre Symbol** (a/p),read symbol of a with respect to p, is defined for integers, a, which are not divisible by p. It is equal to 1 if a is a quadratic residue modulo p and to −1 if a is a quadratic non-residue modulo p.

The **Jacobi Symbol** (a/P), where $P = p_1p_2p_3$.... (notice that some factors may be repeated), and where (a,P) = 1, is defined thus: (a/P) = $(a/p_1)(a/p_2)(a/p_3)$...... for example (219/383) = −(383/219) = −(164/219) = −(41/219) = −(219/41) = −(14/41) = (2/41)(7/41) = −(7/41) = −(41/7) = −(−1/7) = 1 hence 219 is a quadratic residue modulo 383.

NOTE Many simple properties of L & J symbols are illustrated in this numerical example.

Euler's Function, ∅(a) is defined for all positive integers,a, as the number of integers in 0,1,2,...a−1 which are coprime with a. For example, ∅(4) = 2, ∅(5) = 4, ∅(6) = 2.

If a&m are positive integers such that (a,m) = 1 then k is the *smallest* integer such that $a^k \equiv 1(m)$ is called the **exponent** to which a belongs modulo m. For example, 7 belongs to exponent 2 modulo 4.

If, however, k = ∅(m) the a is called a **primitive root** modulo m. For example, 3 is a primitive root to moduli 17, 289 & 578.

The Investigation

Mr Brady is concerned with the 'uniqueness' of the sequence of residues (remainders) r_i, generated when any primitive root g_i of a prime (safe or otherwise) is raised to the power of all of the integers less than p, with reduction modulo p: that is, $r_i \equiv g_i^k$ (p) where $1 \leq i, k \leq p-1$ & $r_i < p$. ....(**)

He observes that EM Burton *et al, Elementary Number Theory*, state that the above operation generates all the residues modulo p in *some* order, not that each primitive root of a prime number generates a unique sequence from all others. There are (p−1)! permutations of the integers less than p, but only a maximum of (p−3)/2 primitive roots, this being attained for safe primes.

To identify the primitive roots of a safe prime number it is only necessary to compute the Legendre Symbols of all the integers less than p−1. Those integers ,i, which are quadratic non-residues, that is, for which (i/p) = −1 are primitive roots.

Project A Write and implement a program to generate safe primes.

Project B Write and implement a program to determine the (p−3)/2 primitive roots of the safe primes from **A** above.

Project C Investigate the sequence generated by (**) above with particular reference to the observation regarding EM Burton *et al*.

Attempts at some, or all, of the above projects may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel (Nebo) 09746-548 to arrive by 1 November 1990.

Review March 1990

This investigation proved to be the most popular since that of Palindromic Numbers in February 1985 . . . why? Many of the submissions have been forwarded to the proposer, Michael Meierruth in Milan. Despite a degree of ambiguity regarding 'odd diagonals' as distinct from 'normal diagonals', efforts worthy of mention include: Paul Cleary, who identified a readily soluble subset (4n+6)(4n+6) reaching 1450 × 1450 in 3.9secs on an Atari 520; and Robin Merson with a most detailed and professionally presented analysis.

The selection of a prize-winning entry has proved to be difficult. But within the spirit of Numbers Count in particular, and empirical number theory in general, it is Antonio Key of 134 Astwood Road, Worcester WR3 8EZ who, having used Quick Basic (v4.5) on an Advent 286/20 for a search program, discarded this having observed a pattern, made an empirical conjecture of an analytic solution and attempted to verify this up to N = 32766. Albeit, it must be said, not for the most general problem.

Details of state-of-the-art and other solutions from Michael Meierruth, Via Treviso 33, 20127 Milan, Italy.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

This month's quickie

No answers, no prizes. Look at the following series of letters:
TELHB
What letter in the alphabet comes next?

Prize Puzzle

There are five consecutively numbered houses in a street, each with a front door of a different colour, and inhabited by men of different nationalities, different occupations, each playing a different sport and each preferring a different drink.

1 The Englishman lives in the house with the red door.
2 The Spaniard's sport is golf.
3 Beer is drunk in the house with the green door.
4 The Ukranian drinks vodka.
5 The number of the house with the green door is one more than the number of the house with the ivory door.
6 The Teacher's sport is tennis.
7 The Accountant lives in the house with the yellow door.
8 The man in the middle house drinks gin.
9 The Norwegian lives in the lowest numbered house.
10 The Solicitor lives in the house next to the hiker.
11 The Accountant lives in the house next to the jogger.
12 The Doctor drinks rum.
13 The Dentist is Japanese.
14 The Norwegian lives next to the house with the blue door.

Now, who drinks whisky, and whose sport is swimming?

Answers on postcards or backs of sealed envelopes — no letters please. Send to: September Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 30 September 1990.

Winner, June 1990

A very good response to the logic problem in the June issue — exactly 125 entries were received, most of which were correct. As usual we had to draw our winner from the heap and the lucky card came from Mr Stephen Lambert of Hull. Congratulations, Stephen, your prize is on its way. To the other 124 entries, keep puzzling, it could be your turn next.

The correct solution was as follows. The pairings were:
Albert-Celia : Barry-Betty : Charlie-Alice.
Albert bought 32 — Celia bought 31
Barry bought 12 — Betty bought 9
Charlie bought 8 — Alice bought 1 ■

A simple partitioning problem, or how to copy from a hard disk sub-directory onto one or more floppies, investigated by Mike Mudge.

The area of investigation this month, which is more on the practical side (or at least almost so!) than those usually to be found in 'Numbers Count', is due to Michael Meierruth of Milan who describes the level of difficulty as 'relatively low'. The background concerns the inadequacy of utility programs to efficiently copy the contents of a hard disk sub-directory onto one or more floppy disks. These all do the obvious of sequentially copying as many files as possible and requesting a change of floppy disk when the next file will not fit in the available space.

The investigation is concerned with working out, and implementing, an algorithm for optimising the copying process: that is, using the minimum number of floppy disks while not requiring an excessive amount of calculation time.

Consider floppy disks having 3600-byte capacity and then ask what is the arrangement of files needed on each floppy disk to accommodate the 17 files below on no more than 3 floppy disks: 63, 127, 175, 190, 215, 311, 407, 463, 517, 537, 711, 801, 827, 1019, 1251, 1272, 1914. This is a simple partitioning problem which, due to its nature, is subject to the time consuming difficulties of typical combinatorial problems. A further complication may also arise if not all the floppy disks have the same capacity, and further if the problem has not been set up so as to fill all of the floppies exactly. (Note: the above problem, being artificial, does indeed fill all of the floppies. However, the odds are very much against this occurring in a practical situation.)

The unique answer to the above problem is:
Floppy 1: 63, 311, 463, 711, 801, 1251. Total 3600.
Floppy 2: 127, 215, 517, 827, 1914. Total 3600.
Floppy 3: 175, 190, 407, 537, 1019, 1272. Total 3600.

This solution was obtained by an exhaustive search, readily seen to require an excessive amount of calculation time when the number of files is large and/or when either of the above-mentioned complications are present.

One possibly promising algorithm involves ordering the files by size and then starting to copy from the side with the largest files. When the next smallest file will not fit on the remaining space, start copying from the side with the smallest files, continuing this process until all files have been copied. This algorithm is seen to fail to generate the optimal number of floppies in the artificial case above. However, Michael Meierruth claims a success rate (defined as those cases resulting in the use of the optimal number of floppies) of about 78% using randomly generated samples, compared with a 30% success rate using the same samples with simple sequential copying. Michael also has a slightly altered version of the above algorithm which yields a 98.5% success rate!

Readers are challenged to reproduce, and if possible improve upon, the above success rates. Particular attention should be given to the simulation (random generation process) of the file samples both with regard to their number and spread of magnitudes. A unit of 1 floppy accompanied by decimal file sizes may be relevant here?

Attempts at this challenge may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel (Nebo) 09746-548 to arrive by 1 December 1990. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded, by *PCW*, to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, together with sufficient details of programs to enable the results (and in particular the random samples of files) to be reproduced. Run times, a summary of results obtained and suggestions for further work in this area should also be enclosed, in a form suitable for publication in *PCW*. Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review April 1990, Pseudoprimes & Carmichael Numbers

This research area attracted considerable attention. Among those submissions deserving of a mention are Norman Meeres, who referred to *Recreations in the Theory of Numbers* by AH Beiler for (A) and then used Basic on the Lynx 48k to find Carmichael numbers up to 49,657. Gordon Mills used an Amstrad PC1640 to investigate (A) up to 29,341 for bases 2/3/5/7 and then to find the lowest Carmichael numbers with up to 15 factors together with complete lists up to 3×10^6 for 3, 4 & 5 factor Carmichaels. Frank Webster's results included 5 & 6 factor Carmichaels less than 2×10^9. Gareth Suggett implemented a routine obtained from Fred Hartley, a previous prizewinner, while Henry Ibstedt used Turbo Basic on a Tandon 386 with a 387 co-processor to provide a graphical and numerical display of prime bases less than 100 which produce simultaneous pseudoprimes for the bases 2,3,5 . . 19; further, a list of all Carmichael numbers less than 5×10^7 was produced in about 140 hours.

However, after much soul searching, this month the worthy prizewinner is David Kirkby of 5 Parkmead, Loughton, Essex IG10 3JW, who used a range of different IBM compatible computers including three 80386 based machines, each with 4Mb of RAM, and an i486 machine witl 16Mb, 'more than one computer often used simultaneously.'

Bases up to 289 for which th smallest pseudoprime is even were investigated and all Carmichaels up to 270857521 found; the largest Carmichael found by David has 75 digits! H has also studied the paper of Jack Chernick, *On Fermat's Simple Theorem*, Bull Amer Math Soc 1939, vol 45, pp 269-274.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

This month's quickie

No answers, no prizes, for the quickie. Arrange four pennies into two straight lines of coins with three pennies in each.

Prize Puzzle

What is the smallest number which:
Divides by 3 with 1 left over.
Divides by 5 with 2 left over.
Divides by 7 with 3 left over.
Divides by 11 with 4 left over.
Divides by 13 with 5 left over.
Divides by 17 with 6 left over.
Divides by 19 with 7 left over.

Answers on postcards or backs of sealed envelopes — no letters please. Send to: October Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 31 October 1990.

Winner, July 1990

A very good response to the July puzzle about the Celtic and Rangers fans, which we said was difficult but in fact turned out to be remarkably easy providing a micro *wasn't* used! Our mistake? We assumed that people would adopt a computer solution, whereas all that was needed was to put the numbe 1-30 round the circumference of a circle and score out ever 13th until 15 had been removed. The solution is to p the Celtic fans in the position 1, 2, 3, 4, 5, 10, 11, 14, 16, 1 19, 21, 24, 27, 28.

Most of the 180 entries we correct and the winning card, which was drawn at random from the pile, came from Mr Andrew Peabody of Doncast Well done, Andrew, your priz will be with you shortly. Usua message to the also-rans — keep trying, it could be your t next.

Part Three of the relationships between quadratic partitions and cubic residues is explored by Mike Mudge.

This is the final part of the investigation of quadratic partitions and cubic residues, originally proposed by NV Meeres of Esher. The first two parts are to be found in *PCW* of June and August 1990 and have already generated considerable interest among readers. This presentation is, however, self-contained; the required mathematical concepts being defined and illustrated in Appendix A.

Throughout this work, p denotes a prime number congruent to unity modulo 6, and hence having the associated quadratic partitions (QPs) of the form $p = 3\times A^2 + B^2$ and $4p = 27\times C^2 + D^2 = 3\times E^2 + F^2$. Now consider the composite qr and ask when this is a cubic residue (CR) of p given that neither of its factors is. Firstly, the case when q is congruent to ±4 modulo 9, for example 5, 13, 23, 31 & 67, and $EF \equiv 0$ mod 9. Then it is found that 4q is a CR but that neither q nor 2q arc. (Table 1.)

If EF or any factor of EF (whether prime or composite) is congruent to ±1 mod 9 it is a CR of p. Reference to the earlier articles will reveal this result for AB in $3\times A^2 + B^2$ and indicate that the factors of AB can be even or odd. However, those factors of EF can only be odd since both E and F are odd. (Table 2.)

The ultimate test of this analysis is whether or not it is capable of generating a complete set of CRs for a given (fairly small) p, hence PROJECT C. Design and implement a computer program which, given as input a prime $p \equiv 1$ mod 6, generates a complete set of cubic residues. (Using any method....) Test case p = 79, analysed by NV Meeres using the results of this series of articles thus: $79 = 3(5^2) + 2^2$; A = 5, B = 2, AB = 10. Thus one pair of CRs (10,69). The least multiple of $B \equiv \pm2$ mod 9 is 8, hence (8,71). $4p = 316 = 27(1^2) + 17^2$; C = 1, D = 17, hence (1,78) & (17,62). The quotient of 1 + 17, on division by 6, is odd, therefore (12,67) are CRs.

Moreover 2,3 & 6 are all non-cubic which shows that 2 & 3 must have indices of the same sign mod 3. Hence $18 = 2\times3^2$ will have an index of the same sign mod 3 as $12 = 2^2\times3$, hence (18,61), and from 10 and 69 in turn (15,64); (46,33); (22,57) & (38,41) are CRs. Also, $4p = 316 = 3(7^2) + 13^2$; E = 7, F = 13, hence (14,65); (21,58); and finally (52,27) complete the set of CRs of 79.

Attempts at some or all of the above projects may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel 09746-548, to arrive by 1 January 1991. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained; together with suggestions for further work in this area.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Table 1: $4p = 3xE^2 + F^2$; $q \equiv \pm4$ mod.9; $2q \equiv \pm1$ mod 9

p	E	F	2q	Cubic Residues
61	5	13	10, 26	20, 52
151	5	23	10, 46	20, 92
367	13	31	26, 62	52, 124
547				
619				
1123				
1483				

PROJECT A: Design and implement a computer program to obtain a completed version of this table and to extend the list of p-values which respond to the above analysis.

Table 2: $4p = 3xE^2 + F^2$; EF or any factor congruent to ±1 mod 9

p	E	F	EF	Cubic Residues	
73	1	17	17	1, 17	
271	19	1	19	1, 19	
409	23	7	161	161	
613	19	37	703	19, 37	**703 is also a CR of 613. Why?**
751					
757					
877					
937					
1609	13	77	1001	14, 22, 52, 91, 308.	**Illustrating all three ways of using E and F**
1951					

PROJECT B: Design and implement a computer program to obtain a completed version of this table and to extend the list of p-values which respond to the above analysis.

APPENDIX A: Some pure mathematical concepts and their definitions.

(1) **Modulus & Congruence** Two integers m & n are said to be congruent modulo a third integer r if and only if (iff) they differ by a multiple of r. That is, m & n leave the same remainder on division by r. We write $m \equiv n(r)$, or $m \equiv n$ mod r, for example $17 \equiv 65(4)$, $17-65 = -12\times4$.

(2) **Quadratic Partition (QP)** A QP of a given integer, K, is simply an expression of K as the sum of multiples of two squares. For example, $68 = 3\times4^2 + 5\times2^2$ is a QP of 68.

(3) **Cubic Residue (CR)** If p greater than 2 does not divide, a, and there exists an integer n such that $a \equiv n^3(p)$, then a is a CR of p. For example, if p = 19 then $11 \equiv 5^3(19)$ and 11 is a CR of p.

(4) **Order** The order of an integer, a, modulo p is defined to be the smallest power of a which is congruent to 1 modulo p.

Review May 1990, Harshad Numbers

This problem, together with the 'Thought for the month . . . On Stanbury Primes' produced many detailed responses. Unfortunately space does not permit a detailed discussion of the results, which are expected to appear in *PCW* December 1990. However, the very worthy prizewinner is Richard Tobin, of 2FR 53 Spottiswoode Street, Edinburgh EH9 1DQ. Using a MIPS RS2030 workstation running at about 12 MIPS Richard found an almost complete list of the first H and NZH numbers for digit sums up to 500 (base 10). The omissions 275, 370, 385, 404 & 495 were attacked for about four days each without result! Any assistance would be greatly appreciated.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for future Numbers Count articles.

LEISURE LINES

JJ Clessa's brainteasers.

This Month's Quickie

No answers, no prizes, for the quickie. In my golf club, 48% of the members are ladies. 25% of the lady members took part in the club tournament and 50% of the gentlemen. What percentage of the total membership took part?

Prize Puzzle, November 1990

A problem in permutations and combinations this month, simple to present and easy to solve — or is it? Get the micros working on it.

If every vertex of a regular octogon is connected with every other, how many triangles will be formed?

Answers on postcards or backs of sealed envelopes — no letters please. Send to: November Prize Puzzle, PCW Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 30 November 1990.

Winner, August

Loads and loads of entries to the August palindromic puzzle, proving that it was far too easy. You just can't win! Still, it wore out a few micros, we suspect. By the way, we received about a dozen late entries for the July puzzle, nine of which were from the same entrant. Really, Mr IH of Glasgow, if you're going to waste money on sending multiple entries, at least make sure they arrive in time! Since this month's winner comes from overseas, there's no excuse for any late entries from the UK. The correct solution(s) were:

1 A & B — 321123
2 B & C — 0110
3 A & C — 111111
4 A & B & C — 655666

and the winning card came from Norway — Mr SA Knudsen of Stabekk. Congratulations, Mr Knudsen, let's hope your prize gets to you OK.

An ill-defined problem in arithmetic, with apologies to the Channel 4 panel game Countdown, presented by Mike Mudge.

This month the area of investigation is believed to be that of ingenious interactive programming and requires no mathematical knowledge beyond the rules of basic arithmetic.

Does the mnemonic BODMAS mean anything to any *PCW* readers; in particular those who were confused in their early days of arithmetic by obtaining incorrect answers from non-scientific calculators?

An Ill-Defined Problem Given a finite set of integers called the base, a set of arithmetic operators (typically $+, -, \times, \div$) and an unlimited number of brackets, construct (if possible) a given target integer.

Any readers who watch 'Countdown' on Channel 4 will realise that the numbers game played there is a special case of this problem; as indeed (usually with a very large set of operators) is the problem of construction of the natural numbers from 1 to N using 3 threes or 4 fours or 5 fives, etc. For example, construct the target integer 784 from (75, 11, 6, 4, 3, 1). A brief thought should yield a solution $784 = 75\times(6+4)+11\times3+1$ . . . but how is that arrived at? Can all positive integers, less than some N, be constructed from the given base set?

With the base set as input, devise a program (probably interactive) to assist in the thought processes identified above particularly applicable to large base sets and associated targets.

Any attempt to rationalise the above nebulous problem may be sent to Mike Mudge, 1 Dolboeth, Cwm Mabws, Llanrhystud, Dyfed SY23 5BB, tel 0974-272548, to arrive by 1 February 1991. Any communications received will be judged, using subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained; together with suggestions for further work in this area, all in a form suitable for publication in *PCW*.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Postscript Further to Numbers Count 86, April 1990: David Kirkby (prizewinner) of 5 Parkmead, Loughton, Essex IG10 3JW, will supply a listing of Carmichael Numbers up to at least 1.3 billion upon receipt of a suitable SAE.

Review, Harshad Numbers May 1990

The topics of Harshad Numbers and Stanbury Primes generated many prizeworthy responses, in addition to that of the prizewinner Richard Tobin discussed last month. Hugh Spence, using Turbo Pascal on a 10MHz Opus PCV, found all 408 NZH for 12 and smallest H-numbers for 10(1)296 except 165, 185, 200 & 275. Michael Rostron used compiled Quick Basic on an Amstrad 2386 to obtain results including H-numbers for bases 2 to 10 with digit sums from the base to 50. Ed Heron, using Forth on an RM Nimbus with an 80186 chip running at 8MHz, found the 408 NZH for 12 in 63 minutes: his investigation of Stanbury Primes found that 'the "formula" was very good up to around a=587 but subsequently deteriorated to about as good as random'! Arnold Bailson concentrated on the Stanbury Primes using HiSoft Modula-2 PC Development System on an RM Nimbus PC186 and found that the ratios No. of Stanbury Primes/No. of Stanbury Numbers and No. of primes/ No. of natural numbers were in 'remarkable' agreement up to 4275328913.

Is there a kind of 'uniform distribution' suggested here? Bruce Halsey found smallest H-numbers for 165:70(17×9's)5, 185:684(18×9's)5, 200:3(2×9's)800 & 275:34 (29×9's)25 and wonders about 297? Jos Pardo used an Olivetti XP9, an 80386 computer running at 33MHz, to investigate Stanbury Primes. Details are available on request.

Review, June 1990

The response to this article, the first of three parts due to Norman Meeres of Esher, suggests that the topic of quadratic partitions and cubic residues has much to commend it. Norman has advanced the concept of 'compatible pairs of primes', these being primes p & q, neither of the form $27a^2 + b^2$ but such that pq is of that form. Then a and b are cubic residues of both primes, for example:

$$7 \times 13 = 91 = 27(1^2) + 8^2$$
$$13 \times 67 = 871 = 27(5^2) + 14^2$$
$$19 \times 73 + 1387 = 27(7^2) + 8^2$$

A preliminary computer-aided survey suggests that about one sixth of the relevant prime pairs are 'compatible' in accordance with the above definition. The distribution of these 'compatible prime pairs' is a topic surely worthy of further investigation.

Frank Webster used BBC Basic on an Acorn Electron to investigate the 1610 primes of the form 6m+1 less than 30,000: A,B,C,D values were found in 33 minutes and projects B & C were then completed.

However, the prizewinner this month is Gareth Suggett of 31 Harrow Road, Worthing, Sussex BN11 4RB, who found that 'this problem didn't really inspire me to great heights' but nonetheless generated a program which, for each prime $6x+1$, lists its complete set of cubic residues ('calculated by brute force'). The relevant A,B,C,D values were then obtained and all of the conditions discussed in the article verified.

An overview of this problem area can be expected in *PCW* for June 1991 when further responses, prompted by the articles for August and November 1990, have been analysed.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and computational mathematics. Particularly welcome are suggestions for further Numbers Count articles.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Another year almost gone again. Here's a couple of puzzles to get you in the mood during the run-up to the holiday period.

This Month's Quickie

No answers, no prizes, for the quickie. A man smoked 100 cigarettes (shame on him) in five days, each day smoking six cigarettes more than the day before. How many cigarettes did he smoke on the first day?

Prize Puzzle

A certain 6-digit number, when multiplied by an integer less than 10, gives a product which is the original number with its digits reversed. There are only two 6-digit numbers which have this property (I hope!), although the integer multiplier is different in each case.

What are the numbers, and what are the respective integers? (Numbers with leading zeros not permitted.)

Answers on postcards or backs of sealed envelopes — no letters please. Send to: December Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 31 December 1990. Good Luck!

Winner, September 1990

We had a record response to the September puzzle, with well over 200 entries coming in. It seems that the logic puzzles are quite popular with our readers. And, as with last month, our winning entry came from beyond these shores — from Mr PB Edmonds of Guernsey, Channel Islands. Not surprising, since we get quite a high percentage of overseas entries every month. Our congratulations go to you, Mr Edmonds, as well as your prize which you should receive shortly.

The correct solution was: the Norwegian drinks Whisky, the Japanese sport is swimming.

To all the also-rans, don't give up. It could be your turn next.