Prime time

Mike Mudge mulls over prime divisors of binomial coefficients.

What are binomial coefficients?
Binomial coefficients are an array of positive integers denoted by ${}_nC_r$ or $\binom{n}{r}$ defined by $\frac{n!}{(n-r)!\,r!}$: and either considered as counting the number of combinations or selections of n unlike things taken r at a time, or the coefficient of $x^r y^{n-r}$ in the expansion in ascending powers of x of $(x + y)^n$. $0 \leqslant r \leqslant n$.

The calculation of ${}_nC_r$ may be carried out using the above definition where, of course, N! denotes the product $1 \times 2 \times 3 \times 4 \times \ldots \times N$ and is read as factorial – N: or using a convenient button on a pocket calculator: or using Pascal's Triangle; thus,

```
      1   1
    1   2   1
  1   3   3   1
1   4   6   4   1 etc.
```

$${}_{n-1}C_r + {}_{n-1}C_{r-1} = {}_nC_r$$

each term (other than the bounding 1's) in a given row being the sum of the two terms immediately above. The row count corresponds to n and the diagonal count to r, thus ${}_4C_2 = 6$.

The calculation of, say, ${}_{44}C_6$ using Pascal's Triangle would require the use of 44 rows (an interesting formatting exercise in Basic!): however,

$$\begin{aligned} {}_{44}C_6 &= \frac{44!}{38!\,6!} \\ &= \frac{44 \times 43 \times 42 \times 41 \times 40 \times 39}{6 \times 5 \times 4 \times 3 \times 2 \times 1} \\ &= 44 \times 43 \times 7 \times 41 \times 13 \\ &= 7059052 \end{aligned}$$

Problem 1 The construction of an efficient algorithm to generate the expansion into prime factors of ${}_nC_r$. Interested readers may consult Pierre Goetgheluck, *American Mathematical Monthly*, volume 94, 1987, pp360–365.

Problem 2 The graphical representation of the distinct prime factors, p, of ${}_nC_r$. *Hint* Using a different set of p, r – axes for *each* value of n, the point (p,r) is plotted if, and only if, p divides ${}_nC_r$.

How does the general shape of this graph evolve as n increases?

Problem 3 Tabulation of w(n,r) the total number of distinct prime factors of ${}_nC_r$.

Problem 3* Erdös has found that w(2n,n) approaches E(n) = n log(4)/log(n) as n becomes very large: obtain empirical evidence for this.

Test data	n	w(2n,n)	E(n)	%Error
	500	116	112	3.45
	1000	208	201	3.37

Problem 4 Erdös conjectured that for all $n > 4$ it is true that ${}_{2n}C_n$ is never square-free: obtain empirical evidence for this. *Note* It is known to be true if (i) $4 < n < 2^{42205184}$ or (ii) $n \neq 2^a$ but verification is an exercise in efficient programming!!

Readers are invited to send their attempts at some, or all, of these problems to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 March 1989.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of results obtained, together with suggestions for further investigation, all in a form suitable for publication in *PCW*.

These submissions will be judged using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date.

Please note that submissions can only be returned if a suitable stamped, addressed envelope is provided.

Review: July 1988

This review will be concerned only with a report of the results obtained by the very worthy prize winner, Gordon Mills of Lyndon House, Catbrook, Chipping Campden, Glos GL55 6DG.

This major computing effort of 'well over 100 hours' using Basic 2 on an Amstrad PC1640 with integer limitation of 2^{31} yielded many interesting results, including those shown in the box below.

Empirical analysis included plotting n against the logarithm of both the minimum sum and minimum product; this casts doubt on the possible results quoted in the box.

Can any readers help Gordon to further extend these results, and possibly publish a paper on this subject?

Minimum sums of n-tuples from 7 to 11 as:

n	sum	product
7	511	1965600
8	1022	15724800
9	1287	34927200
10	2574	279417600
11	5148	2235340800

Possibles include:

n	12	16	20	28
sum	9282	1137708	271085958	315599627733390

Stop Press
AP Bermingham of 25 Murray Mews, London NW1 9RH, has addressed the problem of generating prime numbers through programming in Hypertalk on a Mac II. The two techniques used are: (i) The Sieve of Eratosthenes; and (ii) The Wilson Congruence $(p - 1)! = p - 1$ (modulo p).

How do other readers generate prime numbers?

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles. All letters will be answered in due course.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie
What is the smallest number that uses the vowels A E I O U, and Y, once only, when it is written down? (For example, Twenty Four uses E Y O and U.)

Prize Puzzle
A few months ago we published a quickie in which four 7's were used to generate the successive numbers 1-20. This month's prize puzzle is a variant of the theme.

Using as few 7's as possible, together with the mathematical symbols shown below, generate exactly the value 13579.02468.

Permitted symbols are + – / * ! √. ( ). No others, and only digit 7 may be used. For example, if we'd asked you to generate 100.01 the answer could have been:

$$\frac{77}{.77} + \frac{.77}{77}$$

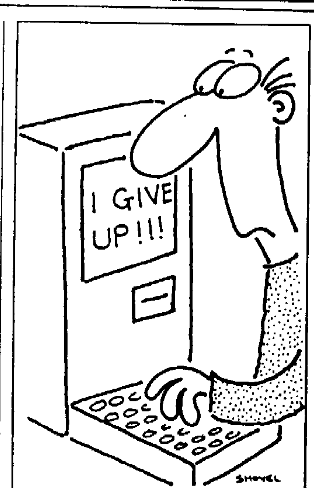
Answers on postcards only, please, to arrive not later than 31 January 1989. Please state clearly how many 7's were used.

Prize Puzzle, October 1988
This problem didn't present too much difficulty when attacked by computer trial and error. Just less than 100 entries were received, with the winning card coming from Mr O'Connell of Shanklin, Isle of Wight. Congratulations, Mr O'Connell, your prize is on its way.

There are six possible solutions, each of which gives 16 occurences. We accepted any, although most readers sent them all in. They are:

1702
2017
2503
2764
3025
3367

To all the runners-up, keep trying — you could be next.



NUMBERS COUNT

Mike Mudge looks at the generation, storage and analysis of Lucas Sequences.

In the latter half of the last century, E Lucas, working in Paris, considered two distinct one-dimensional arrays which we shall denote by $(U_0, U_1, U_2, \ldots)$ and $(V_0, V_1, V_2, \ldots)$. The elements of these arrays are positive integers (or zero) and are defined in two stages:
(i) the first two elements of each array are given, $U_0=0$, $U_1=1$, $V_0=2$ and $V_1=P$.
(ii) Two integer parameters P & Q are specified, and all subsequent elements of the arrays are defined in terms of the two preceding ones by the linear recurrence relations of the second order in terms of P & Q,
$U_n=P\times U_{n-1} - Q\times U_{n-2}$
similarly
$V_n=P \times V_{n-1} - Q\times V_{n-2}$.

We now illustrate particular examples of these Lucas Sequences.

Example I $P=1$, $Q=-1$.
Recall that $U_0=0, U_1=1$ and, therefore, using the given values of P & Q in the recurrence relations above:
$U_2=U_1+U_0=1$, $U_3=U_2+U_1=2$ etc.
$3,5,8,13, \ldots U_{46}=1836311903 \ldots$
These are the Fibonacci Numbers (see 'Numbers Count', *PCW*, May 1983).

For $3 \leqslant n \leqslant 1000$, 21 Prime Fibonacci Numbers are known, $U_3, U_4, U_5, \ldots U_{569}, U_{571}$. What are the other members of this list? Recall that $V_0=2$, $V_1=P1$ and, therefore, using the given values of P & Q in the recurrence relations above:
$V_2=V_1+V_0=3$, $V_3=V_2+V_1=4$ etc.
$7,11,29, \ldots V_{46}=4106118243 \ldots$
These are the Lucas Numbers for $0 \leqslant n \leqslant 500$, 22 Prime Lucas Numbers are known, $V_0, V_2, V_4, \ldots V_{353}$. What are the other members of this list? $V_{503}, V_{613}, V_{617}$ and V_{803} are also known to be prime.

Example II $P=3$, $Q=2$
Exercise for the reader to identify the U_n and the V_n, certainly no computer is needed! Note that $U_{50}=1125899906842623$ while $V_{50}=1125899906842625$.

Example III $P=2$, $Q=-1$. Here it is readily seen that $U_{10}=2378$, $U_{20}=15994428$ and $U_{40}=723573111879672$. These are the Pell Numbers.
$V_{10}=6726$, $V_{20}=45239074$ and $V_{40}=2046573816377474$. These are the Companion Pell Numbers.

Example IV $P=4$, $Q=3$.
$U_{10}=29524$, $U_{20}=1743392200$
$U_{30}=102945566047324$
$V_{10}=59050$, $V_{20}=3486784402$
$V_{30}=205891132094650$
Do these have any particular significance? What are their prime factors?

Detail any other Lucas Sequences which have attracted particular attention in number theory.

Readers are invited to generate Lucas Sequences for particular P & Q values, hence verifying some or all of the numerical results for U_n and V_n given here. Further problems involving the Prime Lucas and Prime Fibonacci numbers, and the identification in Example II and factorisation in Example IV, are hinted at.

Attempts at some or all of these problems may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 April 1989. These submissions will be judged using suitable subjective criteria and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of programs and run times, and a summary of results obtained, together with suggestions for further investigation; all in a form suitable for publication in *PCW*.

Please note that submissions can only be returned if a suitable stamped, addressed envelope is provided.

Note Further to the decoding challenge from TK Boyd in *PCW*, October 1988, the program listing in the box below may help.

Mr Boyd still thinks that without the appropriate seed value a decode is unlikely. Who can prove him wrong?

Review, August

This topic involved Prime Residue Indices and Artin's constant, it was surprisingly popular, and all interested readers are referred to a book called *Repunits and Repetends* by Samuel Yates, particularly pp102-105.

Mention must be made of the efforts in Forth on an Atari 520 STFM with a single-sided disk drive by Christopher Brooksbank of Peterborough; also the 230 hours of BBC Basic on an Electron attacking Problem B up to 15,000 primes by Frank Webster of Middlesbrough.

However, this month's very worthy prizewinner is Fred Hartley of 46 Hughes Road, Hayes, Middx UB3 3AP, who has used an Archimedes 310 in ARM assembler to calculate indices for 11.6 million primes in about 68 hours. Fred has also carried out a very detailed empirical and theoretical analysis of this problem (details on request) and should be recognised in the literature as and when his results are published.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future articles. All letters will be answered in due course.

```
10 INPUT"SEED",S     11 IF S>0 THEN S=S* -1     20 C1=RND(S)
30 INPUT"MESSAGE",M$     35 VDU2     40 FORC1=1TOLEN(M$)
50 C2=ASC(MID$(M$,C1,1))+RND(255)     60 IF C2<256 PRINT"";
62 IF C2<16 PRINT" ";     70 PRINT; ~C2;" "; 80 NEXT     90     VDU3
```


LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie
Here's one to get you thinking. If the letters of each word in an English dictionary are put into alphabetical order (for example, Fish would become FHIS, Cat would be ACT) and the resulting 'words' were then made into an alphabetical list, which original English word would be the last in the list, and which would be the second? (First would be 'A', of course.)

Prize Puzzle
My thanks to Roy Newham of Nottingham for sending in the idea for this puzzle — almost four years ago. (We may be slow but we got there in the end!)

A roll of cloth 120 inches wide is cut into a number of lengths. If every length and every diagonal of each piece cut is a different exact number of inches, what is the maximum length that the roll could be?

Answers on postcards only (or backs of sealed envelopes) to reach us not later than 28 February 1989. Send your entries to:

February Prize Puzzle,
Leisure Lines,
Personal Computer World,
VNU House,
32–34 Broadwick Street,
London W1A 2HG.

Note Winners of the November Prize Puzzle will be announced in next month's issue.

PURSUITS

All mod cons

Mike Mudge investigates the Chinese remainder theorem (CRT).

Definition
A is said to be congruent to B modulo m, if and only if A − B is an integer multiple of m. This relationship is written $A \equiv B \pmod m$ or simply $A \equiv B(m)$. Alternatively, we may say that A and B leave the same remainder when divided by m; the remainder is called the residue of A (or B) modula m. For example, $32 \equiv 17(5)$; also $982 \equiv -143(9)$.

Historical note
A simple example, taken from a whole family of ancient Chinese puzzles, asks for a number which leaves a remainder 1 when divided by 3, a remainder 2 when divided by 5, and a remainder 3 when divided by 7.

A solution is therefore sought to the three simultaneous congruences: $x \equiv 1(3)$; $x \equiv 2(5)$; $x \equiv 3(7)$. . . I

The Chinese Remainder theorem
Let $m_1, m_2, \ldots m_r$ be pairwise relatively prime integers. Then the system of simultaneous congruences: $x \equiv a_1(m_1)$; $x \equiv a_2(m_2)$;. . .$x \equiv a_r(m_r)$ has a unique solution modulo $M = m_1 m_2 \ldots m_r$.

Constructively, if $M_k = M/m_k$ for $k = 1, 2, \ldots r$ and if further y_k is defined by $M_k y_k \equiv 1(m_k)$ then the required solution is $x = a_1 M_1 y_1 + a_2 M_2 y_2 + \ldots a_r M_r y_r$.

For complete proof see, for example, KH Rosen, *Elementary Number Theory and its Applications*, Addison-Wesley 1984.

Example
To solve the three simultaneous congruences I above: $a_1 = 1$, $a_2 = 2$, $a_3 = 3$: $M = 3.5.7 = 105$, $M_1 = 105/3 = 35$, $M_2 = 105/5 = 21$, $M_3 = 105/7 = 15$.

Solve $35y_1 \equiv 1(3)$, thus $2y_1 \equiv 1(3)$ or $y_1 \equiv 2(3)$; similarly from $21y_2 \equiv 1(5)$ find $y_2 \equiv 1(5)$ and from $15y_3 \equiv 1(7)$ find $y_3 \equiv 1(7)$. Hence from CRT $x \equiv 1.35.2 + 2.21.1 + 3.15.1 \equiv 157 \equiv 52(105)$.

Example
To solve the three simultaneous congruences $x \equiv 1(5)$, $x \equiv 2(6)$ and $x \equiv 3(7)$ use $a_1 = 1$, $a_2 = 2$, $a_3 = 3$, $m_1 = 5$, $m_2 = 6$ and $m_3 = 7$ to yield $M = 210$, $M_1 = 42$, $M_2 = 35$, $M_3 = 30$, $y_1 = 3(5)$, $y_2 \equiv 5(6)$, $y_3 \equiv 4(7)$ and hence $x \equiv 206(210)$.

Multi-length addition
Suppose that the word size of a computer is only 100, but that we wish to do arithmetic with integers as large as 10^6. First find pairwise relatively prime integers less than 100 with a product greater than 10^6; say 99, 98, 97 and 95. Convert integers less than 10^6 into quadruplets consisting of their least positive residues modulo the above integers. (**Note** This requires multi-precision operations but is only carried out once for all.)

Finally carry out the required arithmetic operations on the members of the appropriate quadruplets (to the appropriate modulus), and combine the results using the CRT.

Example To add $x = 123684$ and $y = 413456$ on a computer of word size 100 using the moduli suggested above:
$x \equiv 33(99)$, $y \equiv 32(99)$ hence $x + y \equiv 65(99)$
$x \equiv 8(98)$, $y \equiv 92(98)$ hence $x + y \equiv 2(98)$
$x \equiv 9(97)$, $y \equiv 42(97)$ hence $x + y \equiv 51(97)$
$x \equiv 89(95)$, $y \equiv 16(95)$ hence $x + y \equiv 10(95)$. . II

Solving II by CRT using $M = 89403930$, $M_1 = 903070$, $M_2 = 912288$, $M_3 = 921690$, $M_4 = 941094$ yields $y_1 \equiv 37(99)$, $y_2 \equiv 38(98)$, $y_3 \equiv 24(97)$, $y_4 \equiv 4(95)$ hence by CRT, $x + y \equiv 537140$ (89403930) and by 'order of magnitude test' $x + y = 537140$.

Readers who doubt the usefulness of this approach should consult D Knuth, *The Art of Computer Programming: Semi-Numerical Algorithms*, Volume 2 (2nd edition, Addison-Wesley 1981).

Problem
Readers are invited to write computer programs to:
1) Solve systems of linear congruences of the type found in the CRT. Test case: $x \equiv 2,3,4,5\&6$ moduli 11,12,13,17&19. Answer $x \equiv 150999(554268)$.
2) Carry out multi-precision addition and multiplication using the CRT.

Attempts at either or both of these projects may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, by 1 May 1989.

Review, September

Rare and very rare primes generated an above average response, probably due to the omission of the 'square' on the modulus in the Wilson Primes: . . .p_W^2. . . sorry!

Sierpinski Primes: if there are only finitely many then there are infinitely many composite Fermat Numbers. Reg Bond pointed out that the given bound of 3×10^{10} is at least 4.137×10^{10}.

Best general reference once again, *The Book of Prime Number Records* by Paulo Ribenboim, Springer-Verlag 1988. I do have a list of 850 prime numbers with more than 1000 digits constructed by Samuel Yates, if any reader is interested — 15 have more than 10,000 digits.

This month's prizewinner is John C McCarthy of 168 Fairholme Drive, Mansfield, Notts NG19 6DU, whose work shows that success can be achieved in interpretive Superbasic on a QL.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles. All letters will be answered in due course.

Brainteasers courtesy of JJ Chessa.

Quickie
Which is the odd one out in the following set?
HIJACK CANOPY THIRST
MONDAY DEFEAT
STUPID SIGHING
No prizes, so don't send answers.

Prize Puzzle
Very simple this month — perhaps.

What is the smallest number that can be written using only the digits 3,5 and 7 such that both the number and also the sum of its digits are exactly divisible by 3,5 and 7?

Answer on postcards (or backs of envelopes) to: *PCW* Prize Puzzle March, *PCW* Editorial, VNU House, 32-32 Broadwick Street, London W1A 2HG, to arrive not later than last post on 31 March 1989.

Good Luck!

Winners

Because of the Xmas holiday and advanced publishing deadlines, the winners of the November and December prize puzzles are announced in this issue.

November 1988
101 entries — mostly correct. The winner was RT Deadill of Southampton with the correct solution of £47.41 starting wage.

December 1988
The Christmas crossword attracted about 150 entries. The winner was Mr Mark Gill of Abingdon, Oxfordshire and the solution is shown alongside.

Our congratulations to both winners and their prizes are on their way. To all the near misses, keep trying!

1	0	2	0	1	■	2	6	■	8	1	5
0	■	0	■	6	7	6	■	1	4	5	0
1	1	2	■	9	■	0	■	4	1	0	■
■	3	■	2	0	5	■	2	5	■	■	2
1	■	1	3	0	■	2	9	■	■	6	1
0	■	6	1	■	1	8	0	■	5	6	0
1	7	9	■	1	3	0	■	1	2	■	2
5	2	■	■	1	8	■	2	5	7	■	5
0	■	■	1	2	■	1	6	2	■	5	■
■	3	6	0	■	1	■	2	■	3	1	3
1	1	2	0	■	5	1	4	■	3	■	2
1	4	4	■	1	3	■	4	0	8	0	4

Square dances

Mike Mudge investigates 'nearly square' primes and follows up a conjecture of Hardy and Littlewood.

This problem began for me with a letter from Dr Charles Lindsay of 96 Princetown Road, Bangor BT20 3TG. Charles defines a 'Trio' to be two prime numbers and the perfect square with respect to which they are symmetrically placed. Algebraically we can write a Trio as $T=(p_1,n^2,p_2)$ where $n^2-p_1=p_2-n^2=w$, (width of Trio).

A simple investigation reveals Trios thus: (7,9,11); (61,64,67); (79,81,83); . . . (1669, 1681, 1693); (2593, 2601, 2609); . . .

Problem 1 Is there a formula for the n^{th} Trio?

Problem 2 Are there infinitely many Trios?

An attempt to discover the background to this problem was a failure, however it revealed what appears to be a related problem from Hardy and Littlewood, 1923. These most eminent mathematicians conjectured that:

'There exist infinitely many prime pairs of the form (m^2+1, m^2+3). The number of such primes m^2+3 less than n is given asymptotically by:

$$Q(n) \text{ approaching } \frac{3(n)^{1/2}}{(\log n)^2}\, P\left(\frac{p(p-v)}{(p-1)^2}\right)$$

where we use P to denote the product of all terms with p greater than 3, and v is defined by:

$$v=\begin{cases} 0 \text{ when } (-1/p)=(-3/p)=-1 \\ 2 \text{ when } (-1/p)(-3/p)=-1 \\ 4 \text{ when } (-1/p)=(-3/p)=+1.\text{'} \end{cases}$$

Mathematically inclined readers should note that (n/p) is the Legendre Symbol defined as +1 if n is a quadratic residue modulo p and −1 otherwise.

Problem 3 Construct the sequence of prime pairs of the form (m^2+1, m^2+3).

Note These are simply Trios of width 2.

Problem 4 (Mathematics permitting.) Evaluate Q(n) for various n (that is, the upper limit to the search for Trios of width 2) and provide empirical evidence for the asymptotic distribution conjectured above.

Alternative Problem 4 What happens to Charles Lindsay's original concept if squares are replaced by cubes, fourth powers, fifth powers and so on.

In other words, are there generalised trios ('GTs') given by (p_1, n^r, p_2) having width defined by $n^r-p_1=p_2-n^r=$ ^{w}GT?

Attempts at some or all of the above problems may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 June 1989. Any submissions received will be judged using suitable subjective criteria and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of results obtained.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, October

This problem was concerned with the evaluation of a quadratic polynomial with integer coefficients for a sequence of integer argument. Basic question: for how long can the results have modulus unity or be prime? How many of the results are found to have this property if the range is extended beyond the initial one? V(f(x),N) was defined on page 240, *PCW*, October 1988.

The prizewinner this month is Bruce Halsey of 31 Marlborough Green Crescent, Martham, Great Yarmouth, Norfolk NR29 4ST.

Bruce used Fast Basic on an Atari 520ST and offers the following empirical result: 'On looking at my results I think that the highest values of V should be produced by quadratics where a=1, b is an odd number of about −1000 and c odd around 125,000 . . . I would be glad to share my sheaf of results with anyone else interested in the subject.' Take up this offer and let us hear more about V(f)x),N)!

Robin Merson, a regular correspondent to this column, has examined in detail the case N=1000 and finds the optimal solution of 659 primes for (a,b,c) = (1,35,248063) and a sub-optimal solution of 657 primes for (a,b,c)−(1,1,247757).

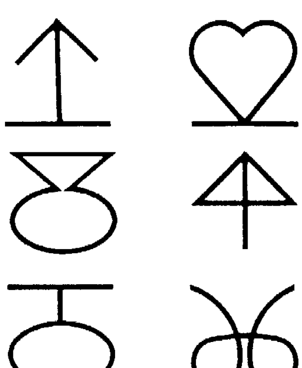
Reg Bond, another 'Numbers Count' stalwart, has recollected his computations of between 1½ and 10 years ago to yield estimation formulae giving approximations to the number of prime values of x^2+x+p for $1 \leq x \leq N$; details on request.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles.

Brainteasers courtesy of JJ Clessa.

Quickie

This month's quickie has been supplied by Master Richard Sparnon of Morecambe who challenges you to find the next member of the following series:



Many thanks, Richard – that should keep a few people guessing!

Prize Puzzle

This month's problem is one of logic, but for those who prefer to use sledgehammer methods, it can be done by micro.

In the following multiplication, each letter represents a digit. What digit is Q?

```
S P R I N T
          Q
-----------
P R I N T S
```

Answers on postcards or backs of envelopes to: *PCW* Prize Puzzle April, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 30 April 1989.

Prize Puzzle, January

A very low response to this problem – only 14 entries were received. It wasn't really a true problem for micro solution, but we did have an outright winner and the random number generator was not needed.

The winning entry came from Mike Waterman of Camberley who receives our congratulations. Unfortunately, Mr Waterman, we could not decipher your address. If you care to drop me a line, you will receive your prize.

Mr Waterman's solution requires only 16 sevens:

7 * [7! +77 +7·7 +7·7] * √(·7 + ·7⁷) +7! −77 −7/ ·7

Untouchable Numbers

Mike Mudge investigates the Aliquot Parts of positive integers, a topic which prompts questions first asked in 1657 and which requires clever use of a computer for verification.

Definition (i) The Aliquot Parts (or factors) of a positive integer are defined to be those positive integers, including unity, which are less than the integer and divide it exactly — that is, without remainder.

Definition (ii) Given a positive integer, n, the sum of its Aliquot Parts shall be written S(n).

Paul Erdös has proved (1979) that there are infinitely many m such that the equation $S(x)=m$ has no solution. Jack Alanen (1972) called such m Untouchable Numbers.

There are known to be 89 untouchable numbers less than one thousand, the sequence begins: 2,5,52,88, . . .

Problem 1 Write a computer program to input a positive integer, n, and to output S(n). Hence determine all the untouchable numbers less than one thousand.

Unsolved problems relating to untouchable numbers include:

a) Is 5 the only odd untouchable number?

b) Are there arbitrarily long sequences of consecutive even numbers which are untouchable?

c) How large can the gap between consecutive untouchable numbers be?

Note In much of the following work, n, will be expressed as the product of its prime factors rather than as an explicit integer, thus 888 will be considered as $2^3.3.37$ and 1444 as $2^2.19^2$.

Readers are therefore encouraged to input, store and output integers in this form and further to address the problem of computing S(n) if $n=p_1^{e_1}.p_2^{e_2}.p_3^{e_3} \ldots p_r^{e_r}$, where the $p_1 \ldots$ to p_r are the distinct prime factors of n, and $e_1 \ldots$ to e_r are their multiplicities. It is not desirable to construct the integer explicitly but rather to choose the possible combinations from the known prime factors.

On 3 January 1657 Pierre Fermat asked for solutions of:

A) $n^3+S(n^3)=m^2$; also of

B) $n^2+S(n^2)=m^3$

On 17 March 1657 John Wallis asked for solutions of:

C) $m^2+S(m^2)=n^2+S(n^2)$

Note With reference to A that $7^3+S(7^3)=7^3+(1+7+7^2)=400 =20^2$; and with reference to C that $4^2+S(4^2)=4^2+(1+2+4+8) =31$ while $5^2+S(5^2)=5^2+(1+5) =31$ also.

A larger and more typical solution of C) due to Frenicle (1658) is given by $n=2.163$, $n^2+S(n^2)=2^2.163^2+1+2+163 +2^2\ 163^2+2.163+2^2 .163+2.163^2=187131$, while $m=11.37$, $m^2+S(m^2)=11^2.37^2 +1+11+37+11^2+37^2+11.37+ 11^2.37+11.37=187131$ also.

Many solutions to A, B and C are known and it is *not* anticipated that readers of *PCW* will add to this body of knowledge.

Problem 2 Write a computer program to input a positive integer, n, in the form of the product of its prime factors and to evaluate, and output, $n^2+S(n^2)$. Hence verify the correctness of the following solutions to C.

n: 3.11.19; 2.5.151; $2^3.3.37$; 29.67; $2^3.7.29.67$.

m: 7.107; $3^3.67$; 2.19.29; 2.3.5.37; 3.5.11.19.37.

Problem 3 Use the program developed in solving problem 2 above to construct m-values given that the following n provide solutions to B.

n: 7.11.29.163.191.439; 43098; $2^2.5.7.11.37.67.163.191 .263.439.499$.

Problem 4 Discover or verify the solution to A given below.

n: $2^5.5.7.31.73.241.243.467$;

m: $2^{12}.3^2.5^3.11.13^2.17.37. 41.113.193.257$;

Attempts at some, or all, of the above problems may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 July 1989. Any submissions received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of results obtained; together with suggestions for further work, all in a form suitable for publication in *PCW*.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, November 1988

The iterative procedure, suggested by Paul Cleary and forming the basis of this article, did not find widespread appeal among *PCW* readers. I wonder why? However, the work of Gareth Suggett of 31 Harrow Road, Worthing, Sussex BN11 4RB deserves the prize award. Gareth claims to have established that sums of squares of factors will always diverge and doubts whether neglecting multiplicities would be sufficient to produce a convergent sequence.

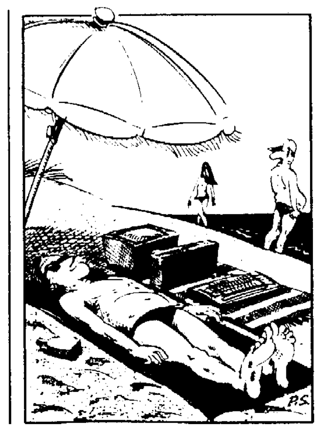
He also suggests a related problem, iterating: $X_n=\mathrm{Int.}(F(X_{n-1}))$ where $F(X)=a_1p_1^y+ a_2p_2^y+ \ldots +a_np_n^y$ in which $X=p_1^{a_1}p_2^{a_2} \ldots p_n^{a_n}$ and y is a real number between 1 and 2.

A recent appeal from Ron J Cook of 113 Critchill Road, Frome, Somerset BA11 4HW, tel: (0373) 64351, has been seen:

'Multiple Precision (or extended) Arithmetic. I understand only the basic principles of this technique . . . I should like to know the principles and structure of these programs . . . a reference to a book or paper, an expert, any lead, I should be extremely grateful . . .' Surely there are many *PCW* readers who can help Ron with this problem.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles. All letters will be answered in due course.

Brainteasers courtesy of JJ Clessa.



Quickie

No answers, no prizes.

When the day after tomorrow is yesterday, today will be as far from Wednesday as today was from Wednesday when the day before yesterday was tomorrow. What day is it now?

Prize Puzzle

There's a certain number X which is the product of 4 different prime numbers (units excluded).

The square of X contains 9 digits, the first three of which are the same as the last three, and the middle three equal the sum of the first and last three (that is, twice the first three). Got it?

What's the largest number that X could be?

Answers on postcards only (or backs of envelopes) to: Leisure Lines Prize Puzzle May 1989, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive not later than 31 May 1989.

Good luck!

Prize Puzzle, February

Only 63 entries for this month's problem, and of these only 46 had the correct solution.

The answer was 11579 inches (or, as some of you insisted on putting it, 321 yards 1 foot 11 inches).

The winning solution came from Anthony Isaacs of London. Congratulations, Mr Isaacs, your prize is on its way. To all the also-rans — keep puzzling, it could be your turn next.

PURSUITS

NUMBERS COUNT

Sequential processing

Mike Mudge invites non-specialist mathematicians to solve straightforward problems, although, courtesy of Douglas Hofstadter, they may not be so simple.

This month's column is a response to many letters I have received, which are typified by: 'your Numbers articles in *PCW* have given me much pleasure over the years, although many of them have been beyond my scope!'

There is no requirement for mathematical training beyond that of second-form algebra in what follows — although the scope for ingenuity of programming, leading to the discovery of new results, is considerable.

Please read on!

Sequence A

$a_1,a_2,a_3,a_4 \ldots$ is a non-terminating sequence of positive integers defined as follows:

$a_1=a_2=1$, $a_n=a_{n-a_{n-1}}+a_{n-a_{n-2}}$ for n greater than 2.

Thus $a_1=1$, $a_2=1$, $a_2=a_{2-1}+a_{2-1}=1+1=2$, $a_3=a_{3-2}+a_{3-1}=2+1=3$, $a_4=a_{4-3}+a_{4-2}=1+2=3$ and the sequence continues 4,5,5,6,6,6,8,8,8,10,9,10,11,11, 12,12,12,12,16,14,14,16,16,16, 16,20,17,17. . .

Problem A Write a computer program to generate this sequence. Do *not* simply list all of the terms computed, but rather address the question: 'Are there infinitely many integers (7,13,15,18. . .) that are missed out?'

Of course a finite computer program cannot provide a yes/no answer to this question, but it will be interesting to see a list of such 'missed out' numbers up to some large value.

Sequence B

$b_1,b_2,b_3,b_4 \ldots$ is a non-terminating sequence of positive integers defined as follows:

$b_1=1$, $b_2=2$, and for n greater than two, b_n is the least integer greater than b_{n-1} which can be expressed as the sum of two or more *consecutive* terms of the sequence.

Thus the sequence begins: 1,2,3,5,6,8,10,11,14,16,17,18,19, 21,22,24,25,29,30,32,33,34,35, 37,40,41,43,45,46,47. . . where, for example, the last term quoted (namely 47) is the least integer greater than the previous term (46) and can be expressed as 1+2+3+5+6+8+10+11. In this case it is the sum of eight consecutive terms of the sequence.

Note that, when determining the sixth term of the sequence, having already obtained 1,2,3,5,6 it is not found to be possible to construct 7 as the sum of two or more consecutive terms and so 3+5=8 yields the next term.

Problem B Write a computer program to generate this sequence. Again study those integers which are missed out.

Sequence C

$c_1,c_2,c_3,c_4 \ldots$ is a non-terminating sequence of positive integers defined as follows:

$c_1=2$, $c_2=3$ and when $c_1 \ldots c_n$ are defined, form all possible expressions c_ic_j-1 where $1 \leq i<j \leq n$ and append them to the sequence, thus obtaining: 2,3,5,9,14,17,26,27,33,41,44,50, 51,53,69,77,80,81,84,87,98,99, 101,105,122,125,129. . .

Notice that to 2,3, we append $2\times3-1=5$; to 2,3,5, we append $2\times5-1=9$ and $3\times5-1=14$, and so on.

Problem C Write a computer program to generate this sequence. Attempt to estimate what fraction of the integers less than 10^n appears in this sequence, for n=1,2,3,4. . .

Attempts at some, or all, of the above problems may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF; tel: (0902) 892141, to arrive by 1 August 1989.

Any submissions received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of results obtained; together with suggestions for further work, all in a form suitable for publication in *PCW*.

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Review, December

The subject of Diophantine equations has occupied the attention of mathematicians over a period of many centuries, starting with the Greeks, Arabs and Indians. Readers requiring an account of the most interesting and important results are encouraged firstly to consult *Diophantine Equations* by LJ Mordell (Academic Press, 1969).

More recent results are distributed throughout the literature of both pure and computational mathematics and can be most efficiently explored using *Mathematical Abstracts* or *The Science Citation Index*.

'Take three cubes': Case 2 has no further solutions satisfying Mod(x+y+z) less than or equal to 150000. Case 5 trivially has no solutions in positive integers. Proof?

Now to $x^4+y^4+z^4=t^4$. Noam D Elkies (*Mathematics of Computation*, volume 51, number 184, October 1988, pages 828-835) initially discovered the solution (2682440, 15365639, 18796760, 20615673) by a direct computer search technique, followed by a second independent solution with digits of the order of 10^{70} *not* by direct search!

However, Noam's work led Roger Frye of Thinking Machines Corporation to find the minimal solution (95800, 217519, 414560, 422481) as reported by Keith Devlin, *Computer Guardian*, May 1988.

This month's prizewinner, for interest in and literature relating to Diophantine equations, is Reg Bond of 75 Laburnum Crescent, Allestree, Derby DE3 2GS.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general of specific, for future Numbers Count articles. All letters will be answered in due course.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie

No prizes — not even an answer for this one. A boy cycles to school at 10mph and returns home at 15mph. What is his average speed for the complete round trip?

Prize Puzzle

Now here's a problem that should please any 'Greens' among our readers. It's about an orchard which originally contained 100 trees set out symmetrically as a 10x10 grid at 10-yard intervals between trees horizontally and vertically.

Each tree was numbered as follows: trees 1–10 were in the first row; trees 11–20 were in the 2nd row; trees 21–30 were in the 3rd row, and so on.

One night a storm arose and, by coincidence, uprooted every tree that was numbered with a prime number — that is, trees numbered 1, 2, 3, 5, 7, 11 and so on — leaving only 74 standing.

Now for the problem. In this depleted orchard, how many different ways can you find four trees which form the corners of an exact square, of any size?

That should get the micros whirring (or the wellies out, for those who prefer to do it manually).

Answers on postcards or backs of envelopes to: Leisure Lines Prize Puzzle June 1989, *Personal Computer World* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive no later than 30 June 1989.

Winner, Prize Puzzle, March

A more-difficult-than-usual problem; some 80-odd entries were received with a few incorrect solutions. The answer:

33 577 577 777 777 775

The winning entry — drawn, as usual, at random from the correct entries — came from Bonnie Scotland, from Mr Graeme Hughes of Glasgow. Congratulations, Graeme — your prize is on its way.

Meanwhile, to all the also-rans, keep puzzling — it could be your turn next!

Factorial functions

A set of five problems involving the factorial function, presented by Mike Mudge.

Definition Given a positive integer, n, the function FACTORIAL n is defined as the product of all of the positive integers up to and including n. We write FACTORIAL n as n! thus: n! = 1x2x3x4x . . . xn. For example 1! = 1, 7! = 1x2x3x4x5x6x7 = 5040.

Problem 1 When is n! expressible as the sum, or difference, of integer powers of a given integer? that is $n! = m^a \pm m^b$. Erdös observed that $n! = 2^a \pm 2^b$ only when n = 1,2,3,4,5. For example $5! = 120 = 128 - 8 = 2^7 - 2^3$.

Problem 2 When is n! expressible as a product of three consecutive integers? That is, n! = (m − 1)(m)(m + 1). Simmons, *Journal of Recreational Mathematics*, vol 1, 1968, p38, found four solutions (m,n) = (2,3), (3,4), (5,5) & (9,6). For example 6! = 720 = 8x9x10.

Problem 3 When is n! + 1 a prime number? That is, n! + 1 = p where p is divisible only by itself and unity. Templer, *Mathematics of Computation*, vol 34, 1980, pp303-304, found eleven solutions: n = 1,2,3,11, . . . 154.

Problem 4 When is n! expressible as a product of two or more non-trivial factorials. That is, n! $a_1!xa_2!xa_3!x \ldots a_r!$ where r and each a_s are greater than one. Hickerson observed:

9! = 7! x 3! x 3! x 2!
10! = 7! x 6! = 7! x 5! x 3!
16! = 14! x 5! x 2!

Problem 5 How close to n! do the integer powers of a given integer become? That is given an integer r find the power m which minimises the modulus of $n! - r^m$.

Croft computed a table for the case r = 2 including:

n	5	20	90
m	7	61	459
d	−1.34	+1.26	−0.0007

Where d is the *percentage* error in the *exponent* m.

Can d be further reduced for a given (or all) n by increasing r from 2 through the values 3,5, . . . etc?

Attempts at some, or all, of the above problems may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF. tel: (0902) 892141, to arrive by 1 September 1989. Any submissions received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of results obtained; together with suggestions for further work, all in a form suitable for publication and a sae.

Review, Jan 1989

This problem related to the factorisation of Binomial Coefficients, a topic first discussed in 'Numbers Count', *PCW* October 1984. A very readable early reference is the paper of P Erdös and G Szekeres, 'Some number theoretic problems on binomial coefficients', *Australian Mathematical Society Gazette*, vol 5, 1978, pp97-99.

However, the most recent paper known to the writer is that of Pierre Goetgheluck, 'On prime divisors of binomial coefficients,' *Mathematics of Computation*, vol 51, no 183, July 1988, pp325-29; which, 'using computational and theoretical methods, deals with prime divisors of binomial coefficients: geometric distribution and number of distinct prime divisors are studied. We give a numerical result on a conjecture by Erdös on square divisors of binomial coefficients.'

Other references which may be relevant include:

A Sarközy, 'On divisors of binomial coefficients I; *Journal of Number Theory*, vol 20, 1985, pp 70-80.
PAB Pleasants, 'The number of prime factors of binomial coefficients'; *Journal of Number Theory*, vol 15, 1982, pp203-225.
P Erdös, H Gupta & SP Khare, 'On the number of distinct prime divisors of $\binom{n}{k}$'; *Utilitas Math* vol 10, 1976, pp51-60.

Worthy responses include:

(1) Michael J Cowan using Basic/Assembler on an Apple IIe with twin disk drives. Michael attacked problem 3* and identified a shortage of memory rather than time: his calculation of w(2n,n) in the case of n = 30000 taking 2.43 hours.

(2) Mathias Meuser used 8080 assembly language on a Bondwell 2 with 2MHz Z80 CPU and 50k of free memory running under CP/M. He considered problems 1, 3* & 4 determining that w(1000,353) = 109 with a largest prime factor of 997 in 25 seconds, while w(30000,10000) = 2039 with a largest prime factor of 29989 was established in 30 minutes.

However, after much thought, this month's worthy prize winner is John Cannell, of 28 The Ridge, Surbiton, Surrey KT5 8HX.

All John's programs are in interpreted Basic running on a BBC Master; a measure of speed being 1.05 seconds for the expansion into prime factors of $\binom{1000}{353}$ compared with 'less than half a second' by Goetgheluck using compiled Pascal.

A feature of John's submission was the graphical display of **(a)** Distinct Prime Factors of $\binom{n}{r}$ and **(b)** Powers of Prime Factors of $\binom{n}{r}$. John showed familiarity with Goetgheluck's work and suggests an alternative to E(n) = n1n(4)/ 1n(n), due to Erdös as a limiting form for w(2n,n) as n becomes very large; that alternative being C(n) = 1.358n/(1n(n) − 0.377).

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general of specific, for future Numbers Count articles. All letters will be answered in due course.

Brainteasers courtesy of JJ Clessa.

Quickie

No answers, no prizes and no apologies from us for this hoary old chestnut:

Three men order a meal in a restaurant. The bill comes to £30, so each man puts down a £10 note as his share. The waiter takes the money with the bill over to the cash desk where the manager has a twinge of conscience and decides he's overcharged the men, so he knocks £5 off the bill — making it £25 for the three meals. He tells the waiter to give the men £5 back. Now the waiter, seeing some easy money, decides to charge the men £27, keeping £2 for himself. So what has happened to the other pound?

Prize Puzzle

This month's problem has been submitted by Anthony Isaacs of London. Many thanks, Mr Isaacs — let's hope the answer you gave me is the correct one!

Using all the digits 0-9 (but no leading zeros), what is the smallest number that can be made which is exactly divisible by every number from 0-18?

Answer on postcards (or backs of sealed envelopes) to: Leisure Lines, Prize Puzzle, July 1989, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG. It should arrive no later than 1 August 1989.

Good Luck!

Prize Puzzle, April

A very simple problem indeed — 165 entries received — but nevertheless, quite a few of these actually had the wrong answer.

The correct answer is Q=3, and the winning entry, drawn at random from the heap, came from Mark Datko of Luton. Congratulations Mr Datko, your prize is on its way.

Meanwhile, to all the alsorans, keep trying — it could be your turn next. This month's puzzle could be your path to fame, glory and riches.

Well, perhaps not quite that, but you could win yourself a beautiful Faber-Castell stainless steel automatic pencil at least!

By the way, we apologise for a misprint in the solution to the January puzzle. The correct answer using 16 7s should have been:

13579.02468 =
7 * [7! + 77 + 7.7 + 7.7] *
$\sqrt{}$ (.7 * .7^7) + 7! − 77 − 7/.7

Sorry about that, and once again let me wish you good luck with this month's problems.

The Maltese Factor

Mike Mudge considers partitions into sums of consecutive integers.

Number of ways	Smallest integer	Prime factors
1	3	3
2	9 = 5+4	3**2
	= 4+3+2	
3	15 = 8+7	3*5
	= 6+5+4	
	= 1+2+3+4+5	
..	..	..
23	3465 = ?	3**2*5*7*11

This problem has been suggested for 'Numbers Count' by Mr Albert Debono, a regular reader of this column living in Malta.

The proposal is to investigate the number of ways in which a given positive integer can be expressed as the sum of consecutive positive integers. For example:

95 = 47 + 48 = 17 + 18 + 19 + 20 + 21
= 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14.

Problem 1 To construct a computer program which, given a positive integer, N, as input would output all the various ways in which N can be written as the sum of consecutive positive integers.

Albert conjectures that the only numbers which will not be expressible in this form are the integer powers of two, that is 2, 4, 8, 16, . . .

Problem 2 To determine the smallest integer which can be expressed as the sum of consecutive positive integers in a given number of ways, and to express these integers in their prime factors. Thus to obtain a table, see the box above.

Investigation of this table suggests the following: 3 is always a factor of the smallest integer that can be expressed as a sum of consecutive integers in n ways? In general, even n generate far larger smallest N than do odd n?

Are these suggestions valid, and if so why?

Problem 3 Extend the above ideas to the representation of a given positive integer as the sum of *consecutive prime numbers*. For example,

100 = 2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23
= 47 + 53.

Attempts at some, or all, of the above problems may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 October 1989. Any submissions received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of results obtained; together with suggestions for further work, all in a form suitable for publication in *PCW*.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Numbers Count Review, February 1989

Among the numerical results requested are:
Prime Fibonacci Numbers U_n occur when n = 3,4,5,7,11,13,17,23,29,43,47,83, 131,137,359,431,433,449,509, 569,571. .2971. . .?
Prime Lucas Numbers V_n occur when n = 0,2,4,5,7,8,11,13,16,17,19,31,37, 41,47,53,61,71,79,113,313,353, . .503. .613. .617. . .803. . .?
Example II Answer $2^n \pm 1$.

A full discussion of this topic is to be found in Paulo Ribenboim's *The Book of Prime Number Records*, Springer-Verlag 1988.

No submission received justifies the award of a prize this month; thus it is proposed to re-open the topic of Lucas Sequences until 1 October 1989 so there is a second chance to submit any relevant material.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles. All letters will be answered in due course.

Brainteasers courtesy of JJ Clessa.

Quickie
No answers, no prizes. Another chestnut — but can you solve it? A brick weighs seven pounds plus half a brick. What is the weight of a brick and a half?

Prize Puzzle
A slightly different problem this month, but one that could get the micros whirring. You are given nine digits and a blank grid. You have to fit the digits so that the clues are matched.

a	b	c
d		
e		

Digits to be used

1 3 3 3 5 7 8 8 9

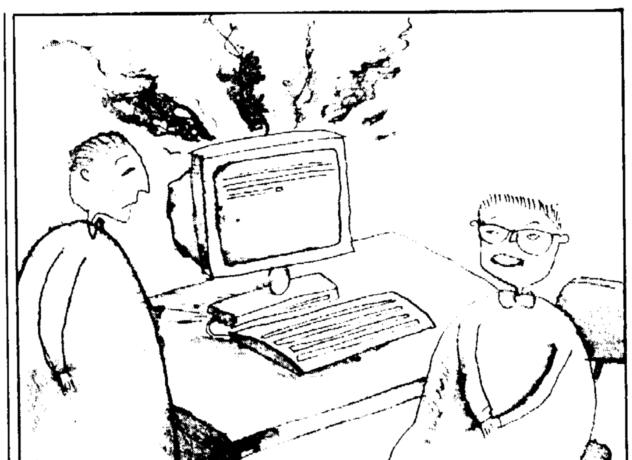
Clues Across
a Multiple of a perfect square
d Digits in arithmetical progression
e Exact multiple of 11

Clues Down
a Palindromic number
b Digits add up to 17
c Prime number

Answers on postcards or backs of envelopes only, please, to arrive by 1 September 1989.

Send your entries to: Leisure Lines Prize Puzzle August, PCW Editorial Office, VNU House, 32-34 Broadwick Street, London W1A 2HG.

Winner May 1989
A smaller than average entry. Exactly 98 entries were received, and several of these had the wrong answer.



'It says it's lost its entire program and can it have a Hamlet cigar.'

The problem is relatively easy if a computer is used, and the required answer is 19019 which is the product of the four prime numbers 7, 11, 13 and 19.

The first correct winning entry, drawn at random from the pile, came from a member of the services — Ms Rhona Knudsen, at BFPO 50 (wherever that is). Congratulations, Rhona, your prize is on its way.

Meanwhile, to the rest of you, keep trying — this month's puzzle could prove to be your turn to win a luxurious Faber-Castell stainless steel automatic pencil. END

NUMBERS COUNT

Mike Mudge investigates Goldbach's Conjecture or 'How can something so simple be so difficult'.

In 1742 the Russian mathematician Christian Goldbach (1690 — 1764) wrote to the Swiss mathematician Leonhard Euler (1707 — 1783) expressing a belief that: 'Every integer greater than 5 is the sum of three primes.' Euler replied that this is easily seen to be equivalent to: 'Every even integer greater than 4 is the sum of two primes.'

Proof left as an exercise for the reader, or see page 229 of *The Book of Prime Number Records* by Paulo Ribenboim (Springer Verlag, 1988).

In 1937 the Russian mathematician IM Vinogradov gave a simplified proof of an earlier result due to GH Hardy and JE Littlewood, 1923, namely that there exists an integer n_0 such that every odd number greater than or equal to n_0 is the sum of three primes. One calculation of n_0 yields $_3 3^{15}$, quite a large number!

Here we leave the theoretical results and examine some empirical results whose reproduction and extension are well within the scope of a typical personal computer.

Problem 1 In his recent book, *Invitation to Number Theory with Pascal* (Camelot Publishing Company, 1989), Donald D Spencer defines a Silverbach Number as an integer that can be expressed as the sum of two primes in three different ways. Thus: 22 = 19 + 3 = 17 + 5 = 11 + 11.
Write a program to generate such numbers, and examine their possible asymptotic frequency.

Problem 2 Spencer also defines a Copperbach Number as an integer that can be expressed as the sum of two primes in two different ways. 18 = 5 + 13 = 7 + 11; 20 = 17 + 3 = 13 + 7
Write a program to generate such numbers, and examine their possible asymptotic frequency.

Problem 3 The alternative title to this month's column is taken from the fascinating book *A Number for Your Thoughts* by Malcolm E Lines, (Adam Hilger, 1986) where a related decomposition is discussed.

Twin Primes are defined to be two prime numbers differing by 2, thus 3, 5; 5,7; 11,13; 17,19; 29,31; 41,43; . . . see *PCW*, July 1984 for a related computational problem discussed in detail in an article entitled 'Brun's Constant' by Ed Rosenstiel.

Lines reports that a computer search of all even integers less than one million found that those greater than 4208 could be represented by the sum of two primes taken from the sequence of twin primes; further, that all even integers larger than 24,098 and less than a million could be expressed as the sum of two twin primes in more than one way, 'Some in more than one thousand different ways.'

Verify the results of Lines and tabulate the number of possible decompositions into the sum of two twin primes for all integers less than one million.

Problem 4 Returning to the Goldbach decomposition into the sum of two primes (for even integers), the question to be addressed is: how to determine the number N above which there are at least n different Goldbach decompositions.

Clearly, the computer search can only examine a finite number of values greater than N but suggested results are in the table below.
Verify these results and generate additional values for the table. In how many ways can 1000000 be decomposed into Goldbach Pairs?

Attempts at some, or all, of the above problems may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 November 1989.

Any submissions received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of results obtained, together with suggestions for further work; all in a form suitable for publication in *PCW*.

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n	50	100	150	600	650	700
N	4688	11672	19246	105368	116618	126878

Suggested results for Problem 4

Review, March

The Chinese Remainder theorem has its earliest known formulation in the Sun Tzu Suan-ching (that is, the mathematical classic of Sun Tzu) which has been dated between 280 AD and 473 AD. In modern times it has formed the basis of 'mind reading' tricks both as party pieces and on the stage. However, readers were asked to consider its application to multi-length, computer-oriented arithmetic.

Several complete implementations were received, together with considerable scepticism regarding its value as a computing tool. Sceptics see D Knuth, *The Art of Computer Programming: Semi-Numerical Algorithms, Volume 2* (2nd edition, Addison-Wesley 1981).

This month's worthy prizewinner is Ed Hersom of Glen Cottage, Bagby, Thirsk, N Yorks YO7 2PF, using Forth on his system which 'has an 80186 chip and an 8087'. Ed gained much inspiration from *Microchip Mathematics — number theory for computer users* by Keith Devlin, which I strongly recommend to all interested readers.

Ed removed the need for the addition of two 'long numbers', which is troublesome, and required only their input, output and the determination of their relevant residues. Altogether a very worthwhile piece of work.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles. All letters will be answered in due course.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Correction, July 1989
Whoops, we goofed! Our apologies to readers for a mistake in the July Prize Puzzle. The problem was stated as follows:

Using all the digits 0-9 (but no leading zeros), what is the smallest number that can be made which is exactly divisible by every number from 0-18?

Clearly the range of divisors of 0-18 is a nonsense, and should have read 1-18.

This month's quickie
No answers, no prizes. You are probably aware of English words in which the vowels a, e, i, o, and u appear in sequence — the word 'facetious' is one of the best known. Can you find a word in which the vowels appear in reverse sequence — that is, u, o, i, e, and a?

Prize Puzzle
Regular puzzlers will, no doubt, be aware of the problem of the cheque in which the pounds and the pence figures are transposed. The standard problem goes something like this:

A lady receives a dividend cheque from her bank manager in which the pounds and the pence values have been transposed. However, unaware of this, she cashes the cheque and spends some of the money (X) before she realises the mistake. She then calculates that at this stage the money she has remaining is an exact multiple (N) of the amount that the cheque should have contained. What should this original cheque have been?

The classic problem then varies depending upon the values of the amount spent (X) and the multiple (N).

In this month's problem, we have six ladies and six transposed cheques. Each lady spends the same sum (X) as the others, and each finds, as before, that she is left with an exact multiple of the intended amount. However, each of these multiples (N) is different. What is the amount spent (X) in each case?

Answers on postcards or backs of sealed envelopes only, to Prize Puzzle September 1989, PCW Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG, to arrive before 1 October 1989. END

NUMBERS COUNT

A little studied constant due to A Ya Khninchin, presented by Mike Mudge.

Some algebraic notation and definitions:

Repeated Summation

$$\sum_{r=1}^{n} a_r = a_1 + a_2 + a_3 + \ldots + a_n$$

$$\sum_{r=1}^{\infty} a_r = a_1 + a_2 + \ldots$$

For example,

$$\sum_{r=1}^{3} 1/r = 1/1 + 1/2 + 1/3 = 11/6$$

$$\sum_{r=1}^{\infty} 1/2^r = 1/2 + 1/4 + 1/8 + 1/16 \ldots = 1$$

8/9 *(geometric series)*

Repeated Multiplication

$$\prod_{r=1}^{n} a_r = a_1 \times a_2 \times a_3 \times \ldots \times a_n$$

$$\prod_{r=1}^{\infty} a_r = a_1 \times a_2 \times \ldots$$

For example

$$\prod_{r=1}^{3} 1/r = 1/1 \times 1/2 \times 1/3 = 1/6$$

$$\prod_{r=1}^{\infty} 1/r = 1/1 \times 1/2 \times 1/3 \times \ldots = 0$$

(Numerator remains 1, while the denominator becomes larger and larger.)

Natural Logarithms
The natural logarithm of a equals b, written 1n a=b, means that b is the power to which the base, e (approx 2.718281828) must be raised to produce b. For example, 1n 1.284 is nearly 0.25=1/4 because (2.718281828)¼; that is, the fourth root of 2.718281828 is nearly 1.284.

The arithmetic mean, A (average) of n quantities, $Q:(a_1,a_2,\ldots.a_n)$ is defined by $A=(1/n)(a_1+a_2+a_3+\ldots.a_n)$.

The geometric mean, G (average) of the same n quantities is defined by $G=n\sqrt{}(a_1 x a_2 x a_3 \ldots.x a_n)$.

For example, if Q:(1,2,3,4,5) then A = (1+2+3+4+5)/5=3 while G=5√ (1×2×3×4×5) approx 2.605.

A continued fraction expansion
Given any positive number, x, the associated continued fraction expansion, $c_0;c_1,c_2,c_3,\ldots.$. is defined as follows:

c_0 is the whole number part of x, c_1 is the whole number part of $1/(x-c_0)$ and so on, thus x=

$$c_0 + \cfrac{1}{c_1 + \cfrac{1}{c_2 + \cfrac{1}{c_3 + \ldots\ldots}}}$$

We write $x=(c_0;c_1,c_2,c_3,\ldots)$ thus a simple calculation shows that 105/38=(2;1,3,4,2).

Definition, Khinchin's Constant, K,

$$K = \prod_{r=1}^{\infty} (1+1/(r^2+2r))^{\frac{\ln r}{\ln 2}}$$

Note carefully that this is the *product* of infinitely many terms, each of which consists of the simple expression $(1+1/(r^2+2r))$ raised to the *power*(1n r)/(1n 2).

Problem (1) Compute, K, to as many *correct* significant figures as possible. Clue — K is approximately 2.6......

Problem (2) Determine the continued fraction expansion of the approximation to, K, which has been calculated in (1).

Warning do not allow the continued fraction algorithm to continue beyond the point where significant figures in, K, are no longer correct.

Problem (3) Investigate the postulate that the geometric mean of the terms in the continued fraction expansion of, K, (neglecting $c_0=2$; that is, the whole number part of, K,) approaches, K, as more of them are included.

Attempts at some, or all, of the above problems may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel (0902) 892141, to arrive by 1 December 1989. Any submissions received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of results obtained; together with suggestions for further work, all in a form suitable for publication in *PCW*.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, April 1989

The problem related to TRIOS and the proposer, Charles Lindsay, provoked only limited response. Unfortunately, the definition given of a TRIO referred to 'two prime numbers' when 'two *consecutive* prime numbers' was intended. However, this point was appreciated by most readers.

Thanks are due to Glenn Taylor for explaining that Problem 2 and alternative Problem 4 are indeed subsets of The Goldbach Conjecture.

Charles Lindsay has broadened the issue with the observations that:
(1) Several TRIOs lie on the same arithmetic progression, so does every (suitable) arithmetic progression contain a (finite?) number of TRIOs?
(2) When an arithmetic progression ceases to have squares mid-way between consecutive primes, do they continue to appear between non-consecutive primes?
(3) It seems likely that all squares are the arithmetic mean of two primes, which are not usually consecutive, so that TRIOs are in some sense 'the tip of an iceberg'.

Comments from readers to myself or Charles directly. This month's prizewinner is John Sutton of 8 Porchester Street, Ascot, Berkshire SL5 9DY. John used his BBC Master to fill (almost) a large ADFS disk. This data was then subjected to various modes of graphical analysis, although no strong conjectures were forthcoming. 'The constant of proportionality in the limit expression for Q seems to lie between 5.5 and 5.7.' Can anyone improve on this?

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LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie
No prizes, no answers, for this one. A girl has as many sisters as she has brothers, but each brother has twice as many sisters as he has brothers. How many brothers and sisters are there in the family?

Winner, July 1989
Probably a bit too easy. There were just over 100 entries, but 20 of these got it wrong (and not because of our goof, which we owned up to in the last issue).

The solution, the smallest number containing all the digits 0-9 which is exactly divisible by every integer from 1-18, is:

2, 438, 195, 760

The winning card drawn at random from the correct entries came from Ms Kathryn Wyatt of Aberdare, Wales, who receives our congratulations and a beautiful prize which will be coming her way shortly.

Prize Puzzle, October 1989
Here's a problem that shouldn't cause you too much grief. Each of the five symbols used in the grid alongside represents a different integer number. However, one of these symbols has been omitted from one square. Using the row and column totals shown, can you deduce which symbol the blank square should contain?

	#	$	$	63
#	$	@	@	50
$	*	&	*	63
#	&	@	#	48
59	61	45	59	

Answers on postcards or backs of sealed envelopes only, to arrive not later than 31 October 1989, to: October Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Street London W1A 2HG.

NUMBERS COUNT

Mike Mudge investigates Greedy Sequences and The Least Integer Solution of the Diophantine Equations.

This subject area has been suggested by a regular 'Numbers Count' correspondent, Peter Cameron of Oxford, and all responses received will be forwarded to him for information.

Definition A 'greedy sequence' is a sequence of positive integers obtained from the natural numbers 1, 2, 3, 4, 5, . . . by imposing a condition to *forbid* certain numbers.

Example A The greedy sum-free sequence contains no three terms s, t, u with s + t = u, where s & t may be equal. Thus 1 is in, 2 is out since 1 + 1 = 2, 3 is in, 4 is out since 1 + 3 = 4, and so on. The sequence of odd numbers is generated.

Example B The greedy sequence in which it is forbidden for any term to be a factor of any other term. 1 must be excluded from the initial sequence or everything else is forbidden! 2 removes all even numbers, 3 removes all multiples of 3, and so on, and the sequence of prime numbers is generated.

Problem (i) The Sidon sequence arises by forbidding x+y = z+w for any four distinct terms x, y, z & w. This initially looks like the Fibonacci sequence (*PCW*, May 1983) but how does it continue? *What is the 10'th term for r = 1, 2 . . . and about how large is the n^{th} term?*

Problem (ii) The MacMahon sequence arises by forbidding any term that is the sum of terms in a subsequence of consecutive terms. This begins 1, 2, 4, 5, 8, 10, . . . but how does it continue? Give a geometrical interpretation of this sequence. Consider question *. . .* above.

Problem (iii) The sequence in which the sum of two distinct terms never divides their product. This sequence, which initially includes most numbers (note, 6 is out since 3+6 divides 3×6) but appears to become less dense, shall be known as the Cameron sequence. Consider question *. . .* above.

Problem (iv) Compute the first 10' terms for r= 1, 2, 3 . . . and hence, or otherwise, find the patterns for the greedy sequences defined by: **a)** no three terms shall be in arithmetic progression — that is, no three terms p, q & r shall satisfy q − p = r − q. **b)** no three terms p,q & r shall satisfy p×q = r — that is, product-free.

The Least Integer Solution of the Diophantine Equations:
$$s = x^3+y^3 = z^3+w^3 - u^3+v^3 = m^3+n^3.$$

This has been computed by E Rosenthal, J A Dardis and C R Rosenstiel using a personal computer and they have proved that
6963472309248 = $2421^3 + 19083^3 = 5436^3 + 18948^3 = 10200^3 + 18072^3 = 13322^3 + 16630^3$
is the least positive integer which can be represented as the sum of two cubes in *four* different ways.

The least positive integer which can be represented as the sum of two cubes in *three* different ways was found by Leech J, *Trans Camb Phil Soc* v53, part 3, July 1957, pp778-780.
87539319 = $167^3 + 436^3 = 228^3 + 423^3 = 255^3 + 414^3$
and is listed in Wells D, *The Penguin Dictionary of Curious and Interesting Numbers*, London, Penguin Books, 1988, p189.

The least positive integer which can be represented as the sum of two cubes in just *two* different ways is 1729 = $9^3 + 10^3 = 1^3 + 12^3$ which is of course the substance of the tale regarding Ramanujan's taxi. See for example Rankin F A, *IMA Bulletin*, 1987, v23, p149.

Attempts at some, or all, of the above problems, together with comments upon or extensions to the result of E Rosentiel *et al*, may be sent tc Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Sout Staffordshire WV4 5NF, tel (0902) 892141 to arrive by 1 January 1990. Solutions to the problems received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to th 'best' contribution arriving by the closing date. It would be appreciated if such submissions contained a brief description of the hardware used, details of programs, run times and a summary of the results obtained, together witl suggestions for further work, all in a form suitable for publication in *PCW*. Please note that submissions can onl be returned if a suitable stamped addressed envelope is provided.

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LEISURE LINES

Winner, August 1989
Another not too difficult problem — or at least, so it seemed to almost 200 entrants. However, about 25% got the wrong anser. One letter from overseas consisted of three pages of logic proving why the problem had no solution!

Many people didn't realise that digits in arithmetical progression could be in descing as well as ascending order. The correct solution — which is unique — is:

a 8	b 3	c 3
d 7	5	3
e 8	9	1

The winning card, drawn as usual randomly from the correct entires, came from Scotland — Mr Neil Jarvis of Edinburgh. Well done, Neil — your prize is on its way.

Meanwhile, to the also-rans — keep trying, it could be your turn next. But a reminder: don't send your solutions in letters — we have to disqualify them. Postcards or backs of sealed envelopes only, please. Also remember to tell us your name and address — there's always one or two entrants who forget to do this. There have been a few occasions on which the card drawn at random had no name and address on it, so someone missed a prize. Don't let it be you.

Prize Puzzle
This month's problem is a pathfinder puzzle. It can certainly be solved by computer if you can write the program to do it.

Starting at square a1 at the top left corner of the 7x7 grid shown, move one square at a time down or to the right until you reach square g7 at the bottom right-hand corner. Add up the numbers in each square you enter — including the first and last squares — to arrive at a total score for the path.

The object is to find the maximum and minimum scores possible, respectively. The solution is unique in each case.

Send the two answers on a postcard or the back of a sealed envelope to arrive not later than 30 November 1989. Send to: November Prize Puzzle, *PCW* Editorial, VNU House, 32-34 Broadwick Stree London W1A 2HG.

a:	3	7	6	7	1	1	4
b:	8	2	9	5	9	4	5
c:	6	9	3	9	5	1	8
d:	5	4	6	5	6	8	8
e:	5	8	3	9	6	0	3
f:	5	6	5	2	8	4	2
g:	9	0	4	6	0	9	9
	1	2	3	4	5	6	7

NUMBERS COUNT

Permutations leading to some famous integer sequences, or 'Have you done any anagrams recently?', from Mike Mudge.

Definition A permutation of n distinct objects is defined to be any particular arrangement of them. (Note: ordering is of fundamental importance in an arrangement, which is why a combination lock should strictly speaking be called a permutation lock, and why any anagram can, in principle, be solved by constructing a complete listing of all possible permutations of the letters involved.)

Since in any particular arrangement (permutation) of n distinct objects there is a choice of 1 from n for the first position, 1 from n-1 for the next position, 1 from n-2 for the next position and so on, with finally 1 from 1 from the n^{th} or last position, it follows that a total of

$n \times (n-1) \times (n-2) \times \ldots \times 1 = n!$

(factorial n) permutations are possible.

Notation Suppose that the n distinct objects are represented (named or labelled) by the positive integers 1,2,3,4....n. A particular permutation may then either be specified by a table in natural order or by a sequence of cycles. Thus the ordering 3,5,4,1,2,6,7 of the first seven positive integers is represented either by the natural order table:

1 2 3 4 5 6 7
3 5 4 1 2 6 7

or by the sequence of cycles

(1 3 4) (2 5) (6) (7),

both of which mean that when starting from the natural ordering replace 1 by 3, 3 by 4, 4 by 1, 2 by 5, 5 by 2, 6 by 6 and 7 by 7.

Stirling Numbers of the First Kind, S(n,k) are defined to be the number of permutations of n distinct objects containing exactly k cycles. For example, if n = 3 the possible 3! = 6 permutations are:

(123);(132);
(1)(23);(2)(13);(3)(12); and
(1)(2)(3).

Hence S(3,1) = 2, S(3,2) = 3 and S(3,3) = 1.

Stirling Numbers of the First Kind

n/k	1	2	3	4	5	6	Row Sum =n!
1	1						1! = 1
2	1	1					2! = 2
3	2	3	1				3! = 6
4	6	11	6	1			4! = 24
5	24	50	35	10	1		5! = 120
6	120	274	225	85	15	1	6! = 720

Subfactorials or Recontres Numbers, R_n are defined to be the number of permutations of n distinct objects in which every object is moved from its natural (or original) position. Such a permutation is technically called a derangement.

Thus R_4 = 9 is displayed in tabular form as follows:

2143, 2341, 2413, 3142, 3412, 3421, 4123, 4312, and 4321.

n	2	3	4	5	6.....
R_n	1	2	9	44	265.....

Euler Numbers, E_n are defined to be the number of permutations of n distinct objects whose associated integers first rise and then alternately fall and rise. (This becomes irrelevant if the objects are considered to be the positive integers themselves — that is, the latter are not merely labels.)

Thus E_4 = 5 is displayed in tabular form as follows:

1324, 1423, 2314, 2413 and 3412: each permutation shown exhibits the required rise, fall, rise . . . pattern of behaviour.

n	1	2	3	4	5	6
E_n	1	1	2	5	16	61

The odd Euler numbers are called tangent numbers, $T_n = E_{2n-1}$, because

$\tan x = 1\ x/1! + 2 \times x^3/3! + 16\ x^5/5! \ldots$

The even Euler numbers (frequently called simply the Euler numbers) should, by analogy, be called secant numbers since

$\sec x = 1 + 1\ x^2/2! + 5\ x^4/4! + 61\ x^6/6! \ldots$

Problem (1) Write a computer program which, by constructing and counting the appropriate permutations, obtains S(n,k), R_n & E_n.

Problem (2) By examining the algorithm/ output of (1) above and obtaining an algebraic generating function or otherwise, obtain an optimal algorithm for generating and factorising S(n,k), R_n & E_n.

Attempts at the problem and/or thoughts on the 'Stop Press' question may be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel (0902) 892141, to arrive by 1 February 1990. Any communications received will be judged, using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution arriving by the closing date.

Review, June 1989 — Hofstadter Sequences

Among the responses to this problem, that of Frank Webster's BBC Basic on an Acorn Electron deserves special mention. Values of a_n up to n = 30000 in 16mins. Conclusion: infinite number of randomly distributed absent values, 5000 terms of b_n in 75hrs. Suggestion of count of missing numbers asymptotic to a straight line representing an increase of 21 missing numbers per 1000 increase in search. Those c_n less than 20000 found in 22hrs; suggestion that number of primes less than N is a major factor in determining the number of missing numbers.

A Parry using MacPascal on a MacPlus then BBC Basic on an Archimedes A310 found the first 145,000 terms (a_n) and 100000' terms (b_n).

The very worthy prizewinner is Anthony Quas of Woodcroft, Weston-in-Gordano, Bristol BS20 8PZ, using 68000 assembler and APL 68000 on a 2Mbyte Atari ST running Mirage. Up to ^{a}200000 in 10 mins. Proof that a_n/n tends to ½ and that a_n/n − ½ could be regarded as a normally distributed random variable . . . Up to ^{c}6000000 in 20 mins. Further that the fraction of the integers occurring tends to about 0.54. A very detailed piece of work, full details from Mike Mudge or Anthony Quas.

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LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Winner, September 1989

We had the problem this time — not you. We omitted to put sufficient constraints on the puzzle and it turned out that more than a thousand solutions satisfied the rules as stated — 1609 to be exact, our most reliable entrant tells us. (Thanks, SNH.)

Probably as a result of this confusion, the total entry was quite a bit down from usual. The winning card came from Mr Bruce Halsey of Norfolk who sent us several of the solutions possible — including the right one, which was £4.95. Congratulations Mr Halsey, your prize is on its way.

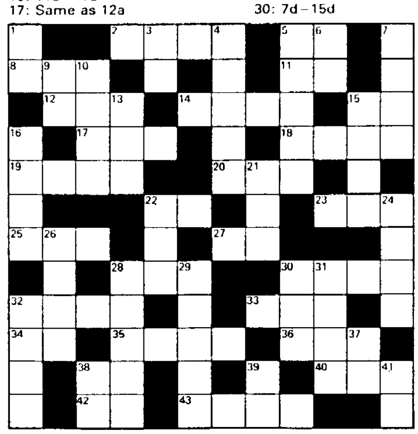
Prize Puzzle, December 1989

This month's problem should be attempted when you are full of turkey, booze and Xmas festive spirit, and need something to keep your brain alive. It's a number crossword. You may use your micro to help you, but you'll probably find that it's not really necessary.

Stick the completed grid to a postcard or to the back of a sealed envelope — not in a letter please — and send it to arrive not later than New Year's Eve 1989, to: December Prize Puzzle, *PCW* Editorial, VNU House, 32–34 Broadwick Street, London W1A 2HG.

Clues Across

2: 5a squared
5: Half of 13d
8: 1d squared
11: 3d−1d
12: One third of 14a
14: 19a with digits rearranged
15: 11a−41d
17: Same as 12a
18: 10 times 25a
19: 2a plus 20a
20: 27a squared
22: 1d−6d
23: 5 times 15a
25: 28a+39d
27: Same as 15a
28: 21d+40a
30: 7d−15d
32: 30a−37d−15d
33: 30d+100
34: 39d+41d
35: 2a+14a+1d+16d
36: Last three digits of 31d
38: One third of 11a
40: One third of 25a
42: 38a times 8
43: Square of 42a



Clues Down

1: One third of 9d
3: Reverse of 1d
4: 4 times 2a
5: 40a squared
6: 9d−3d
7: 10 times 12a
9: Square root of 10d
10: Twice 19a
13: Square root of 4d
15: 10 times 40a
16: 1d times 5a
21: 40 less than 20a
22: 40 more than 13d + 15a
24: 18a−16d
26: 16d+7d
28: 1d times 2a
29: Square of 22d
30: 37d squared
31: 7d−9d
32: 39d squared
37: 27a plus 2
38: One quarter of 34a
39: 34a−41d
41: 5a−11a

Note:
All answers are whole numbers.
28a = the answer to 28 across.
16d = the answer to 16 down.