NUMBERS COUNT

Mike Mudge's subject matter this month goes back to a 20-year-old conference at Oxford University. Why not get stuck in, as he dives into the fascinating world of 'Integer Bases'?

It is first required to define a sequence of positive integers using a simple rule, which may not necessarily be an explicit algebraic formula. Here are some typical sequences which will be used later:

(i) $f(n) = n^2 + 1$ (2, 5, 10, 17, 26, 37, . . .)

(ii) $f(n) = n^2$ (1, 4, 9, 16, 25, 36, . . .)

(iii) Prime Numbers (2, 3, 5, 7, 11, 13, . . .)

(iv) Pseudoprimes* (2, 3, 4, 5, 6, . . .)

(*See *A Handbook of Integer Sequences* by NJ Sloane (Academic Press 1973.))

(v) Primes squared (4, 9, 25, 49, 121, 169, . . .)

(vi) Triangular Nos** (1, 3, 6, 10, 15, 21, . . .)

(vii) $f(n) = n^3$ (1, 8, 27, 64, 125, 216, . . .)

(viii) $f(n) = n^3 + 1$ (2, 9, 28, 65, 126, 217, . . .)

(*See 'Numbers Count', *PCW*, August 1987, page 210.)

Let $S \equiv (s_1, s_2, s_3, \ldots s_k, \ldots)$ denote a general sequence of positive integers; assumed to be non-terminating. P(S) consists of the set of all positive integers which can be expressed as a sum of a finite number of *distinct* terms from S. S is defined to be a 'complete sequence' if, and only if, all sufficiently large integers belong to P(S). That is, there must be a largest integer which does *not* belong to P(S). This integer is called 'the threshold of completeness' of S and is denoted by T(S). For example, for the sequence defined as (i) above, $f(n) = n^2 + 1$, the largest integer which cannot be expressed as a sum of distinct elements of S is 51, we write T(S) = 51.

The corresponding numbers for sequences (ii) . . . (viii) are 128, 6, 1, 17163, 33, 12758 and 8293.

Essential completeness Given a general non-terminating sequence of positive integers S (of which eight particular examples are listed above) we now define associated truncated sequences $S_r \equiv (s_r, s_{r+1}, s_{r+2}, \ldots s_k, \ldots)$ where the r^{th} truncated sequence associated with S is formed by omitting the first r — 1 elements from S. Thus, for example (i) alongside $S_3 \equiv (10, 17, 26, \ldots)$ and it is found that the threshold of completeness for this is $T(S_3) = 255$.

A sequence S is defined to be 'essentially complete' if, and only if, all of its associated truncated sequences S_r are complete.

Theoretical importance. The theoretical importance of this work is centred upon the ratio $T(S_r)/s_{r-1}$; that is, the ratio of the threshold of completeness of the r^{th} truncated sequence to the largest term omitted in its formation.

There is a conjecture that this ratio does not exceed 3 for the sequence (iii) of primes, nor does it exceed 5 for the sequence (ii) generated by n^2.

These conjectures are supported by the empirical evidence shown in Fig 1.

Problem. Readers are invited to produce computer programs to determine the threshold of completeness of a given sequence and hence to evaluate a_{n-1} and obtain, in addition to the results given above (which are intended to provide test-data for debugging and optimising routines) corresponding tables of a_{n-1} for the sequences (i), (iv) . . . (viii). Conjectures similar to those above will be welcome together with theoretical proof of their validity! Submissions should be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Staffordshire WV4 5NF to arrive by 1 April 1988.

All submissions will be judged using subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date.

It would be appreciated if such submissions contained a brief summary of results obtained, in a form suitable for publication in *PCW*.

Please note that submissions can only be returned if a suitable sae is provided.

(ii) $f(n) = n^2$.

n	50	100	150	200	250
s_n	2500	10000	22500	40000	62500
$T(S_n)$	17072	60928	129184	222208	339968
a_{n-1}	7.110	6.216	5.818	5.611	5.483

(iii) Prime Numbers

n	100	500	1000	2000
s_n	541	3571	7919	17389
$T(S_n)$	1683	10779	23859	52247
a_{n-1}	3.217	3.028	3.017	3.004

when $a_{n-1} = T(S_n)/s_{n-1}$

Fig 1

Cyprian's Last Theorem, July 1987

Several contributors found the following results on sums of cubes:

$6^3 = 3^3 + 4^3 + 5^3$
$20^3 = 11^3 + \ldots + 14^3$
$40^3 = 3^3 + 4^3 + \ldots + 22^3$
$60^3 = 6^3 + 7^3 + \ldots + 30^3$
$70^3 = 15^3 + 18^3 + \ldots + 34^3$
$180^3 = 6^3 + 7^3 + \ldots + 69^3$.

Sums of squares yields only Pythagorean Triads discussed fully on other occasions. Searches for powers up to the seventieth failed to yield other solutions.

Dissection of the 6-cube into 3−, 4− and 5− cubes by John Cook in Australia claimed a result with 15 sections, 'one of which was rather irregular' (a letter to the editor from John explaining this would be appreciated!) Richard Tindell discovered an eight piece dissection and later a second in Lindgren's book entitled *Geometric Dissections* (page 24) and attributed to RE Wheeler in *Eureka* (1951).

There is no known dissection into fewer than eight pieces (I hope) but the number of distinct dissections into eight pieces is not clear to me.

However, this month's prize goes to Jonathan Hart of 72 Clifton Rd, Tunbridge Wells, Kent TN2 3AT, who, in addition to dissections of the cube (in fact into twelve pieces), carried out substantial investigation, both theoretical and empirical, on the remaining problem.

As a follow up to this work the Reverend D Cyprian Stockford asks whether integer solutions exist of: $p^2 + q^2 = r^2$ simultaneously with $p^3 + q^3 + r^3 = s^3$? Is this related to the historically famous: $p + q = n^2$ simultaneously with $p^2 + q^2 = m^4$? Before a search is started note that p = 1061652393520 and q = 4565486027761 is, in fact, the smallest solution.

Mike Mudge welcomes correspondence on any subject within the areas of numbers theory and other computational mathematics. Particularly welcome are suggestions, either general or particular, for future Numbers Count articles; all letters will be answered in due course.

Isolated readers can be put into contact with others sharing the same interests. However, greater efficiency regarding published problems should result from contacting the prizewinner.

DIARY DATA

A look ahead at computer shows to May 1988. Readers are advised to check details before setting out on their journey.

OFFICE UPDATE NEC, Birmingham — Andrew Centre (01) 891 5051 Ext 285	**19-22 January 1988**
WHICH COMPUTER? SHOW NEC, Birmingham — Cahners, Belinda Caver (01) 891 5051	**19-22 January 1988**
AMSTRAD COMPUTER SHOW Alexandra Palace, London — Database Exhibitions (061) 456 8383	**29-31 January 1988**
COMPUTERS IN RETAIL AND RETAIL TECHNOLOGY NEC, Birmingham — Focus Events (01) 834 1717	**15-17 March 1988**
ELECTRON & BBC MICRO USER SHOW UMIST, Manchester — Database Exhibitions (061) 456 8383	**18-20 March 1988**
ELECTRONIC PRINTING AND PUBLISHING EXHIBITION Olympia, London — BED Exhibitions (01) 647 1001	**22-24 March 1988**
COMPUTERS IN TRANSPORT AND DISTRIBUTION Wembley Conference Centre, London — Computers in Transport and Distribution (0303) 45979	**19-21 April 1988**
ATARI COMPUTER SHOW Alexandra Palace, London — Database Exhibitions (061) 456 8383	**22-24 April 1988**
COMFEST '88 Telford Exhibition Centre, Telford — (0952) 505522	**12-14 May 1988**

NUMBERS COUNT

Mike Mudge moves his 'number theory' into the practical world of chess.

This month it is assumed that readers are familiar with the basic modes of travel of Queens, Knights, Rooks and Bishops during a chess game.

The Challenge

The problems to be considered, while soluble with a set of very small positive integers, require considerable 'logic' for their efficient analysis together with ingenuity to display any solutions obtained and, finally, inspiration to find a general algebraic theory to explain what is happening.

Problem I. How many Queens?

What is the minimum number, f(n), of Queens that can be placed upon an n x n chess board (the standard board being 8 × 8) so that no Queen is guarding (watching) any other Queen, and also so that the entire board is being guarded (watched) by at least one Queen?

Partial solution

n 5 6 7 8 9 10 11 12 13 14 15 16 . . .

f(n) 3 ? ? 5 ? ? 5 ? ? 8 ? ? . . .

Problem II. How many pieces?

What is the minimum number of pieces of the same type that can be placed upon a standard (8 × 8) chess board so that every square is guarded (watched) by at least one piece?

Partial solution

Queens

q(8) = 5

Knights

k(8) = 12

Bishops

b(8) = ?

Rooks

r(8) = ?

Note The condition that no piece is guarding any other piece is not part of this problem. It is satisfied by the Queens and Bishops but not by the Knights. What about the Rooks?

Problem III

Extend problems I & II above to a general size of board.

Problem IV

Display the set of all (distinct)✶ solutions graphically (or algebraically if no suitable graphics are available) at each stage in I, II & III above.

(✶Equivalent solutions are related one to another either by a rotation of the board or by reflection in a straight line.)

Problem V

Attempt to construct explicit algebraic formulae for Q(n), q(n), k(n), b(n), or r(n): thereby avoiding the need for the logical analysis used above.

How would the graphical (or algebraic) display be produced if indeed a function value for a given n was known?

Problem VI

Consider the extension of problems I to V to 3D chess.

Readers are invited to send their attempts at some or all of the above problems to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel (0902) 892141, to arrive no later than 1 May 1988.

It would be appreciated if such submissions contained a brief summary of results obtained, in a form suitable for publication in *PCW*. These submissions will be judged using subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date.

Please note that submissions can ony be returned if a stamped addressed envelope is provided.

Review: August '87

This produced an acceptable spectrum of response. There was general agreement that the complete solution of (i) is given by: 55, 66, 666. The solution sequence for (ii) begins 1 4, 19600,74909055 . . . for (iii) 1 210, 40755, 7906276, 1533776805 . . . and for (iv) 1, 40755, 1533776805 . . .

Part (v) is fascinating. Using the notation $t_a \pm t_b = t_c t_d$ it is found that:

a 6 18 37 44 86 91 116 132 247 278 392 613 637 662 798 . . .
b 5 14 27 39 65 54 104 125 242 209 374 459 350 275 714 . . .
c 8 23 59 108 106 156 182 346 348 542 766 727 717 1071 1153 . . .
d 3 11 25 20 56 73 51 42 49 183 117 406 532 602 356 . . .

(due to Gareth Suggett).

However, within the spirit of 'Numbers Count' this month's prizewinner is Martin Sann of The Bothy (Home Farm), Firbeck Hall, Firbeck, Nr Worksop, Nottinghamshire, who was attracted to the sequence 1, 36, 1225, 41616, 1413721, 48024900, 1631432881, 55420693056, 1882672131025, 63955431761796, 217260200-7770041, 73804512832419600 . . . of square numbers which are also triangular.

Martin was predicting that the 15th number in this sequence would appear on his BBC 'sometime early in the 22nd century' . . . only to discover subsequently that Rev Canon DB Eperson (then of Bishop Otter College, Chichester) in *The Mathematical Gazette* (Vol 47, page 237, 1963) provides a simple algorithm for generating terms of this sequence. The observation of 'Numbers Count' (*PCW*, March 1983) that only five tetrahedral numbers are also triangular is worth repeating together with the result, first proved by GN Watson in 1918, that only three tetrahedral numbers. are also square. What are the numbers referred to in these results?

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Isolated readers can be put in contact with others sharing the same interests. However, greater efficiency regarding published problems should result from contacting the prizewinner.

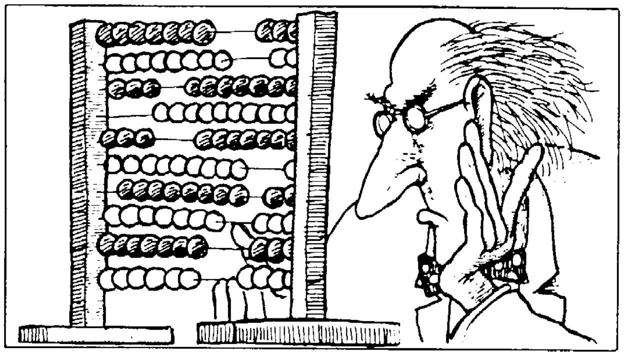
DIARY DATA

A guide to forthcoming computer shows. Readers are advised to check details before setting out on their journey.

ELECTRON & BBC MICRO USER SHOW UMIST, Manchester — Database Exhibitions (061) 456 2991	**18–20 March 1988**
ELECTRONIC PRINTING AND PUBLISHING EXHIBITION Olympia, London — BED Exhibitions (01) 948 9900	**22-24 March 1988**
COMPUTERS IN RETAIL AND RETAIL TECHNOLOGY NEC, Birmingham — Focus Events (01) 834 1717	**29–31 March 1988**
COMPUTERS IN TRANSPORT AND DISTRIBUTION Wembley Conference Centre, London — Computers in Transport and Distribution (0303) 45979	**19–21 April 1988**
ATARI COMPUTER SHOW Alexandra Palace, London — Database Exhibitions (061) 456 2991	**22–24 April 1988**

NUMBERS COUNT

Mike Mudge returns to the popular topic of prime numbers including reference to recently published results.



Definition Denote by p(n) the number of prime numbers not exceeding n. Thus p(1) = 0, ( ); p(10) = 4, (2,3,5,7); p(100) = 25, (2,3,5,7, . . . 79,83,89,97).

The state of the art

EDF Meissel (*Math Ann* 1870, vol 2, pp636–642; 1871, vol 3, p525; 1885, vol 25, pp251–257) calculated and published p(10) = 664579, p(10^8) = 5761455 and p(10^9) . . . which remained the largest published result until 1959. In that year DH Lehmer (*Illinois Journal of Math* vol 3, pp381–388) corrected p(10^9) (the value calculated — by hand — by Meissel being too small by 56) and published p(10^{10}) (in fact, too large by 1).

In 1986 P Shiu (*Math Comp* vol 47, pp351–360) published p(10^{11}) and p(10^{12}) while JC Lagarias, VS Miller and AM Odlyzko (*Math Comp* vol 44, pp537–560) calculated p(4×10^{16}) using approximately 30 hours processing time on an IBM 3081 Model K.

It is clear, therefore, that *PCW* readers should not feel encouraged to extend the range of values of p(n) beyond 4×10^{16}.

Problem

The computing problem associated with p(n) which follows is formulated in such a way that it tests the skill and ingenuity of the programmer rather than the speed and word length of the computer, the efficiency of the compiler or the choice of language.

How many basic operations do *you* need to compute p(n) for a given n? In particular, for n = 10, 100, 1000, 10000. *Note* It is recommended that an algorithm is detailed, coded and checked, then an operation count carried out. If possible, the fundamental operations of arithmetic should be separated into + − * & / and, in turn, separated from logical operations. It is thought inadvisable to attempt this count from the algorithm at its pencil and paper stage. Readers may feel differently! If such a count seems too laborious, an alternative measure of efficiency may be supplied in the form of ratios of times taken to evaluate p(10^n):times taken to evaluate p(10^{n-1}) as a function of n.

As and when multi-precision arithmetic becomes essential, many readers will feel that they are excluded from entry . . . but rest assured an efficient algorithm developed within the normal arithmetic precision of the computer is likely to remain efficient when combined with suitable arithmetic multi-precision routines which may not be immediately available.

Changing the subject:

A Nearly Pattern involving Palindromic Squares

In February 1985 a study of palindromic numbers (reading the same way backwards and forwards) produced the record ever response to a 'Numbers Count' article. Thus it seemed appropriate to quote the result (see Fig 1) of JKR Barnett (*Bulletin IMA*, vol 23, Nos 6/7, June/July 1987 pp100–101).

9	$= 3^2$
94249	$= 307^2$
942060249	$= 30693^2$
9420645460249	$= 3069307^2$
94206450305460249	$= 306930693^2$
942064503484305460249	$= 30693069307^2$
9420645034800084305460249	$= 3069306930693^2$

Fig 1

Now construct the eighth member of the sequence of squares appearing on the right-hand side. It is nearly, but not quite, palindromic! Why?

Readers are invited to send their attempts at eighter, or both, of the above problems to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, to arrive by 1 June 1988.

It would be appreciated if such submissions contained a brief summary of results obtained, in a form suitable for publication in *PCW*. These submissions will be judged using subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date.

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Mike Mudge introduces readers to the elementary concepts of cryptology in this month's 'not-so-secret' Numbers Count.

The need for secret communication in diplomacy and military affairs is readily appreciated. Now that electronic mail, electronic banking and other computer-based business transactions are part of everyday life, the need for security of information is clear to us all. The purpose of this article is to indicate certain aspects of number theory which are the foundations of elementary ciphers (or codes), to display examples of their use, and to invite readers to submit a working coder and decoder package with a specimen message.

It must be emphasised that the types of cipher discussed are elementary and bear little relation to those used in an ultimate security environment; however, they can form part of a challenge among, for example, computer club members: 'How do you go about cracking (even an elementary) code?' This aspect will not be considered here, but may form the subject of a future column, depending upon the response to this article.

Character ciphers

Stage 1 Translate the letters of the alphabet into their numerical equivalents 1–25.

Stage 2 Transform the numerical equivalent, m, of each letter in the message into another number, c, using an 'affine transformation' of the type: c = am + b (modulo 26) where a and b are integers having no common factor. Note that since modulo 26 means retain only the remainder after division by 26, it follows that c lies between 0 and 25 inclusive.

Stage 3 Return each c to its equivalent letter using the reverse process to that described at stage 1; and group into convenient, ordered sets, say, of five to yield the code.

The particular affine transformation in which a = 1 is called a 'shift transformation' and clearly corresponds to replacing each letter of the message by that found by shifting b places through the alphabet.

For example, under the affine transformation c = 7m + 10 (modulo 26), the message 'PLEASE SEND MONEY' becomes the code 'LJMKG MGXFQ EXMW'!

Block ciphers

Stage 1 Group the letters of the message into convenient, ordered sets — say, for example, pairs. For example: 'PLEASE SEND MONEY' becomes 'PL EA SE SE ND MO NE Y'.

Stage 2 Transform the numerical equivalent, m_1m_2, of each pair in the message into another number pair, c_1c_2, using a pair of affine transformations of the type: $c_1 = a_1m_1 + b_1m_2$ (modulo 26), $c_2 = a_2m_1 + b_2m_2$ (modulo 26).

Stage 3 Return each c_1c_2 to its equivalent letter pair using the inverse translation process. For example: 'STOP PAYMENT' block ciphered in triples using the affine transformations:

$c_1 = 11m_1 + 2m_2 + 19m_3$ (modulo 26)

$c_2 = 5m_1 + 23m_2 + 25m_3$ (modulo 26)

$c_3 = 20m_1 + 7m_2 - m_3$ (modulo 26)

becomes 'ITN NEP ACW ULA'.

Exponentiation ciphers

Invented in 1978 by S Pohlig and M Hellman (see *IEEE Transactions on Information Theory* (vol 24, 1978, pp106–110)) this begins by translating the letters of the message into numerical equivalents, using A,B,C, ... Y,Z becomes 00,01,02, ... 24,25. The resulting numbers are then grouped into blocks of '2s' digits; where 2s is the largest positive even integer, such that all blocks of numerical equivalents corresponding to s letters (viewed as a single integer with 2s decimal digits) is less than an odd prime p. Associated with p is the *enciphering key* k, a positive integer which has no common factors with p − 1.

For each message block M, which is an integer with 2s digits, form a code block C using the transformation:

$C = M^k$ (modulo p), O C p.

For example, if p = 2633 and k = 29, then to encipher: 'THIS IS AN EXAMPLE OF AN EXPONENTIATION CIPHER', first convert to two-digit numerical equivalents, then group in blocks of size four: 1907 0818 0818 ... 0704 1723. The final 23 being an X added to complete a block of four.

Now use $C = M^{29}$ (modulo 2633) to obtain the code: 2199 1745 1745 ... 1841 1459.

Readers are invited to send an encoder, a decoder and a specimen message to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, or phone (0902) 892141 by 1 July 1988.

It would be appreciated if such submissions contained a brief description of the enciphering theory and any peculiarities of the programming, in a form suitable for publication in *PCW*. These submissions will be judged using subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date.

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Review: October '87

Space restrictions prevent a detailed review of a very popular topic. Refer to Don Thomasson, *Computing Today*, January 1984, pp52–53, and to this month's worthy prizewinner, Bill Hamley, of Church Lane, Scotter, Gainsborough, Lincolnshire DN21 3RZ.

John Gale of Hemel Hempstead is to be congratulated on his n-dimensional graphics on an Amstrad PC1512 SD, invoking the recursive powers of Pascal. Details on request.

Factorisation of Fermat Numbers. Review, September 1987

The factorisation of fermat numbers, $F_m = 2^{2^m} + 1$, proved to be a very difficult exercise even with the assistance of Theorem 3 (*PCW*, September 1987, page 214).

The table shown here is due to Professor Wilfrid Keller of the University of Hamburg and summarises the state of the art at 1980. This table accompanied the then new results that $1985 \times 2^{933} + 1$ is a factor of F_{931}, $19 \times 2^{6838} + 1$ is a factor of F_{6835}, while $19 \times 2^{9450} + 1$ is a factor of F_{9448}.

Values of m	Character of F_m
0, 1, 2, 3, 4	Prime
5, 6, 7, 8	Composite and completely factored
12*	Four prime factors known
10*, 11*, 19, 30, 36, 38, 150	Two prime factors known
9*, 13*, 15, 16, 17, 18, 21, 23, 25, 26, 27, 29, 32, 39, 42, 52, 55, 58, 62, 63, 66, 71, 73, 77, 81, 91, 93, 99, 117, 125, 144, 147, 201, 207, 215, 226, 228, 250, 255, 267, 268, 284, 287, 298, 316, 329, 416, 452, 544, 556, 692, 744, 931, 1551, 1945, 2023, 2456, 3310, 4724, 6537, 6835, 9448	Only one prime factor known
14	Composite but no factor known
20, 22, 24, 28, 31, 33, 34, 35, etc.	Character unknown

*Cofactor known to be composite

Subsequently GB Gostin and PB McLaughlin (*Math Comp* vol 18, No 158, April 1982 pp645–649) published a new prime factor for each of F_{39}, F_{36}, F_{99}, F_{147}, F_{150} and F_{201}. It is certain that further results exist in the literature and readers are invited to comment on any which they can locate.

Using the flexibility of the 'subjective criteria' this month's prizewinner is Andrew Slodkiewicz of 29 Taylors Road, St Albans, 3021 Victoria, Australia.

Andrew uses string handling routines in Turbo Pascal to manipulate numbers up to 256 digits. Unfortunately his hardware is undefined; however, the calculation of Euler Number E_{152} having 238 digits (for definitions see *PCW*, January 1987) took in excess of four hours to calculate. 'String division is performed using multiple subtractions, then shifting the numerator to the left, and so on. It takes about three seconds per unit in each decimal place.'

Readers may like to write to Andrew with advice or to obtain further details of his work in this area.

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Mike Mudge explains the concept of difference tables.

Many readers will already be familiar with the concept of difference tables. These tables arise in any introduction to numerical methods or, more simply, in the process of interpolation — central to the use of tabulated function values (now, alas, frequently replaced, with a consequent lack of understanding, by the use of the pocket calculator!).

Suppose that $y = f(x)$ is tabulated at equal increments, h, in the independent variable x; these x-values being denoted by x_0, $x_1 = x_0 + h \ldots x_n = x_{n-1} + h = x_0 + nh$ and the corresponding y-values by $y_n = f(x_n)$.

The first forward differences, dy, of y are defined by $dy_n = y_{n+1} - y_n$.

The second forward differences, d^2y, of y are similarly defined by $d^2y_n = d(dy_n)$.

This apparently elaborate algebraic notation is readily clarified by the following example. Suppose $y = x^3 + 1$ with $x_0 = 2$ and $h = 3$: the difference table begins as shown in Fig 1.

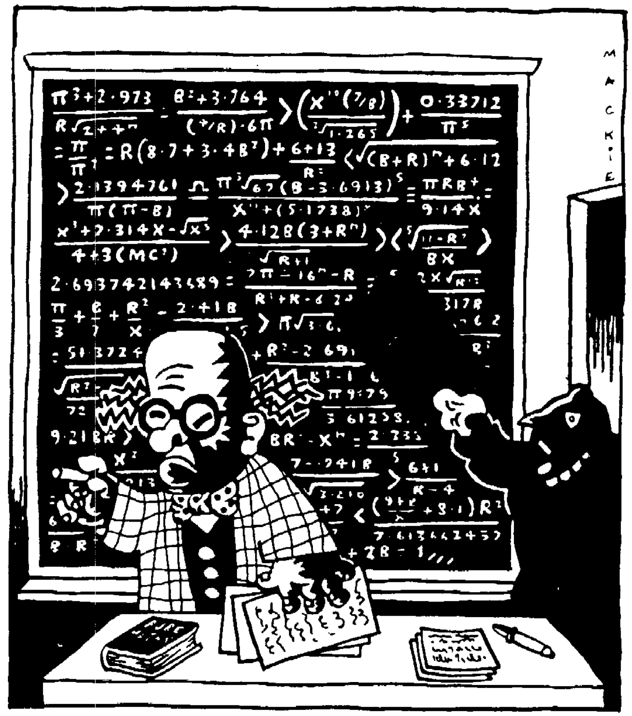
Clearly, the second differences of n^2 are constant and equal to 2.

Question Do there exist non-consecutive integers x_0, $x_1, x_2, \ldots$ such that the second differences of their squares are constant? Specifically, can that constant be equal to 2?

Answer Yes! For example (6, 23, 32, 39) see Fig 3.

Duncan A Buell, of the Supercomputing Research Center, 4380 Forbes Boulevard, Lanham, Maryland 20706, USA, has recently (1987) completely characterised such sequences of length 4 but states that the existence of such sequences of length 5 (and above) is still an open question.

He poses an intermediate step, which he calls problem B; seeking a sequence of five integers n_0^2 n_1^2 n_2^2. N, n_4 where n_0, n_1, n_2 are not consecutive such that their second differences are constant, say, c, and specifically with c = 2.



x	$y = x^3 + 1$	dy	d^2y	d^3y
2	9			
		126 − 9 = 117		162
5	126		270	
		513 − 126 = 387		162
8	513		432	
		1332 − 513 = 819		162
11	1332		594	
		2745 − 1332 = 1413		162
14	2745		756	
		4914 − 2745 = 2169		
17	4914			

Fig 1

n	$y = n^2$	dy	d^2y
1	1		
		3	
2	4		2
		5	
3	9		2
		7	
4	16		2
		9	
5	25		2
		11	
6	36		

Fig 2 The difference table for n^2

i	n_i	$y = n_i^2$	dy	d^2y
0	6	36		
			493	
1	23	529		2
			495	
2	32	1024		2
			497	
3	39	1521		

Fig 3

Problems

(i) Construct a computer program to input function values and print out, correctly formatted, the associated difference table up to the n^{th} differences.

(ii) Search for sequences of four squares such as (6,23,32,39) and (39,70,91,108) whose squares have second constant differences.

(iii) Extend (ii) to sequences of five integers in the pattern of Buell above.

(iv) Attempt to resolve Buell's open question regarding sequences of five squares.

(v) Given that the n^{th} difference of a table of n^{th} powers is constant (see d^3y for $y = x^3 + 1$ above) investigate sequences of non-consecutive integers whose cubes have constant third differences, and so on, through fourth and fifth powers.

Readers are invited to send their attempts at some, or all, of the above problems to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel (0902) 892141, to arrive by 1 August 1988. It would be appreciated if such submissions contained a brief description of the program and a summary of the results obtained in a form suitable for publication in *PCW*.

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Review, November

This problem produced a variety of responses, the largest powerful number seen being 467 9307774, degree 10, base 10. The geometrical interpretation hinted at in the article may well be a figment of the author's imagination — no-one made significant progress along these lines!

The very worthy prizewinner is Brian Stuart of Düsseldorferstr 11, 8000 Munchen 40, West Germany. Brian searches for powerful numbers for all number bases from 3 to 99 to all possible degrees, with a restartable algorithm. By 24 January 1988 he had reached 3×10^6 for all bases and 10^8 for some; with a target of 2^{31} 'at some 11 million per hour'.

Among the many interesting results were: (a) 19 5 16 base 24 (=11080 decimal) is powerful of degree 3 and the only powerful number base 24 less than 119×10^6; and (b) no powerful numbers found to base 90.

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Isolated readers can be put in contact with others sharing the same interests. However, greater efficiency regarding published problems should result from contacting the prizewinner.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie

Here's a message in code. Each letter corresponds to one letter from the original message. If I tell you that the letter 'A' in the coded version equals 'Z' in the original, can you decode it?

GFCKPQ D EJM EJFC DM MEL AJJ, D NDM MFCB MJJB JNN D NDGLG CLG NFC NLA.

Better still, just tell me the meaning of 'QCKP' — it's something you'll probably do when you've decoded the message.

Prize Puzzle

My thanks to Tim Higgins for the idea behind this puzzle.

The other day, the building society sent me notification of interest due on my account. Feeling affluent I rushed out and bought some booze to celebrate — spending an exact number of pounds in the process.

However, when I looked again at the letter, I realised I had mentally transposed the pounds and the pence, and the sum I was due was less than I'd thought.

In fact, I calculated that the amount I had thought I was getting, less the amount spent on booze, was an exact multiple of the amount that I actually received. And coincidentally, this multiple was the same as the pounds that I spent on booze.

If you've managed to understand all that, please tell me how much money I actually received, and how much I spent on booze.

Prize Puzzle, March 1988

First, a word about the April Quickie. I have already received many letters advising me how to use four 7s to generate a value of 26. But, alas, none of them fulfil the conditions of '. . . using standard mathematical symbols . . .'.

Most of you used 'log' which is not a symbol, or [], !! (double factorial), which are certainly not standard symbols. One reader, a maths teacher, even added a couple of extra digits (if that were permissible, Mr W, then $2 \times 77/7 + 7 - 3$ would have been simpler than your effort). The best — but not acceptable — was $7 + 7 + 7 + 7$ which is 26 in base 11 arithmetic.

So, although I'm still hoping that someone will come up with the goods, I'm beginning to doubt it.

Anyway, to the Prize Puzzle and the root of the Fibanacci sequences. The answer, by sheer number crunching, is 144 and 298. Of the 160-odd replies, 151 were correct. The lucky winner, drawn at random, was one of our regular entrants who, I believe, has won before — Mr Alan Northcolt of Winnersh, Berkshire.

NUMBERS COUNT

The fascinating topic of addition chains is explored by Mike Mudge.

Definition An 'Addition Chain' for a positive integer n is a finite sequence of positive integers:

$1 = a_0 < a_1 < a_2 < a_3 < \ldots < a_r = n$ where each member (other than $a_0 = 1$) is the sum of two earlier, but not necessarily distinct, members of the sequence.

Thus, two different addition chains for 14 are:

C_1: 1, 1 + 1 = 2, 2 + 2 = 4, 4 + 2 = 6, 6 + 2 = 8, 8 + 6 = 14

C_2: 1, 1 + 1 = 2, 2 + 2 = 4, 4 + 2 = 6, 4 + 4 = 8, 8 + 6 = 14

Each of these chains is said to have *length*, r = 5.

Definition The minimal length of an addition chain for n is denoted by L(n). A 'Brauer chain' is one in which a shortest chain exists where each member uses the previous member as a summand.

Note that C_2 above is not a Brauer chain because 4 + 4 = 8 does *not* use the previous term — that is, the 6 — but it is a minimal chain.

Any number n which has a Brauer chain is called a 'Brauer number'.

Definition An addition chain for which there is a subset H of the members, such that each member of the chain uses the largest element of H which is less than the member, is called a 'Hansen chain'.

Note that C_2 above is a Hansen chain with H = (1,2,4,8).

Donald Knuth, in *The Art of Computer Programming Vol 2* (Addison-Wesley 1969, pp398-422) gives the following addition chain for 12509:

1, 2, 4, 8, 16, 17, 32, 64, 128, 256, 512, 1024, 1041, 2082, 4164, 8328, 8345, 12509.

This is *not* a Brauer chain since 32 does not use 17. However, it *is* a Hansen chain with H = (1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 1041, 2082, 4164, 8328, 8345).

No Brauer chain of length 17 or less exists for 12509.

A conjecture of Arnold Scholz (1937)

The minimal length of an addition chain for $2^n - 1$ differs from the minimal length of an addition chain for n by less than n.

$L(2^n - 1) \leq n - 1 + L(n)$.

A question of Richard Guy (1983)

Are there any numbers n which do not have Hansen chains? That is, are there any Non-Hansen numbers?

Note the Scholz Conjecture has been proved for $n = 2^a$, $2^a + 2^b$, $2^a + 2^b + 2^c$ and $2^a + 2^b + 2^c + 2^c$ by Utz, Gioia *et al* (1953) and demonstrated for $1 \leq n \leq 18$ and n = 20,24 and 32 by Knuth & Thurber (1973/76).

Problems

(i) Construct a computer program to obtain all possible addition chains for a given n. The complete output should only be generated for certain small values of n for test purposes!

(ii) Modify the above program to list any Brauer and/or Hansen chains produced. In the latter case, the appropriate subset H should be output.

(iii) Establish the value of L(n) as a function on n and hence verify the Scholz conjecture, albeit for a small range of n.

(iv) Comment upon the empirical evidence for the existence of Non-Hansen numbers.

Readers are invited to send their attempts at some, or all, of the above problems to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 September 1988. It would be appreciated if such submissions contained a brief description of the program and a summary of the results obtained in a form suitable for publication in *PCW*. These submissions will be judged using subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date.

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Review: December 1987

Attempts to investigate the sequence directly are clearly doomed, x_{17} having 2661 digits. However, H Ibstedt determined that x_{42} with 89288343500 digits would, if printed 80 digits per line and 60 lines to a page, require more than nine million sheets of paper and weigh approximately 35000kg!

But the most comprehensive study was that of H Ibstedt of 4 Rue Gramme, Paris 75015, whose results include the location of the first non-integer term for all powers up to the eleventh and initial values x_1 from 2 up to 11. A very worthy prize-winner.

The longest integer sequence of 600 terms occurs for cubes and $x_1 = 11$, while the shortest of 7 occurs several times in the above study.

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LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie
No prizes, no answers.

A certain young lady I know refuses to reveal her age. However, she did say that if you multiply the two digits of her age together, and double the result, the answer you get is one more than her age. How old is she?

Prize Puzzle
Rather easy, this one.

I bought a book the other day and discovered that a sheaf of pages were missing. I added up the numbers of the pages that were missing and the total came to 3500.

What are the missing pages?

Answers on postcards only, please, to arrive not later than 31 July 1988.

Send your entry to: Leisure Lines Prize Puzzle — June, Personal Computer World, VNU House, 32-34 Broadwick Street, London W1A 2HG.

Prize Puzzle, April
First, a word about the four 7's quickie. I still do not have a solution to '26' and am convinced that it can't be done. Those who claim solutions invariably use logs or the operator 'e', or introduce symbols, which can hardly be described as standard.

So, it looks like '25' is the limit — one reader quotes Rouse-Ball (*Mathematical Recreations and Essays*) which also states that '25' is the maximum.

Now to the problem that I set for April. It was a long-winded job by computer in most cases, and the total number of different integers which could be made was 9858. The value in the 5000th position was 4381633.

The winner is Mr H McCormick of Sheffield. Congratulations, Mr McCormick, your prize is on its way.

NUMBERS COUNT

Mike Mudge examines JG Triples, primitives and problems.

Definition: Two or more triples of positive integers are said to be JG Triples (the notation arising from the Latin 'Jusdem Generis' meaning 'of the same kind') if they have the same sum and the same product. For example (14, 50, 54); (15, 40, 63); (18, 30, 70); (21, 25, 72) have a common sum equal to 118 and a common product equal to 37800 and thus constitute a quadruple of JG Triples.

Note If (a, b, c) and (p, q, r) are JG Triples then so are (ka, kb, kc) and (kp, kq, kr) where k is any positive integer. Hence we imply the term 'primitive' when referring to n-tuples of J G Triples, there being no common factor throughout all of them.

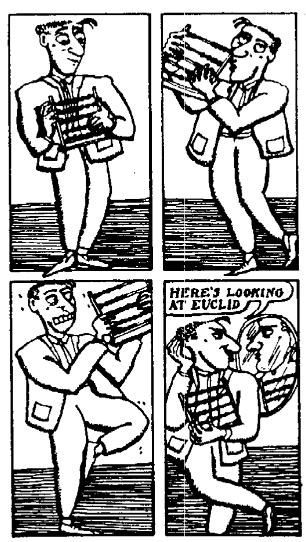
James G Mauldon asked how many different J G Triples exist (c.1979). For quadruples the smallest common sum is 118 (illustrated above) while the smallest common product appears to be 25200 arising from (6, 56, 75); (7, 40, 90); (9, 28, 100); (12, 20, 105). The only quintuplet known to the writer (6, 480, 495); (11, 160, 810); (12, 144, 825); (20, 81, 880); (33, 48, 900) with common sum 981 and common product 1425600.

However, Lorraine L Foster and Gabriel Robins (*American Mathematical Monthly*, vol 89, 1982, problem E 2872) quote results including the least sum for a quintuplet as 185 and for a sextuplet as 400. Also, a decuplet with common sum 1326000 and common product given by $2^7.3^6.5^4.7^2.\ 13^3.\ 17^3$. Further, they conjecture that there are infinitely many primitive n-tuples of JG Triples for any n greater than 4.

Note An infinite family of primitive quadruples of JG Triples is generated by (16ka, bc, 15d); (10ka, 4bc, 6d); (15kb, ad, 16c); (6kb, 4ad, 10c) where a = k + 2, b = k + 3, c = 2k + 7 and d = 3k + 7.

Problems
(i) Construct quadruples, quintuplets and so on of JG Triples verifying in the process the results quoted, in particular those of Foster and Robins regarding minimal sums.
(ii) Conjecture concerning the behaviour of the minimal sum, the minimal product and the number of n-tuples of JG Triples less than a given N_0.
(iii) Investigate the natural extension from JG Triples to JG Quadruples.

A reference to *PCW*, July 1983 and December 1983 may prove helpful. Any other references are rather difficult to obtain.



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Review, January

The apparent abstract nature of this problem deterred many readers. However, an examination of the paper by Shen Lin, *Computational Problems in Abstract Algebra*, Pergamon Press 1970, pp365-370 will reveal the wealth of empirical results obtained by the author nearly 20 years ago and also provide references for background reading. The most readily available is JL Brown: Note on a complete sequence of integers (*American Mathematical Monthly* vol 68, 1961, pp557-560).

One computer program was received which successfully implemented the associated theory and produced T(S), in principle for any sequence de-

ENDZONE

NUMBERS COUNT

fined algebraically, though it must be said that slowness of execution limited the results obtained to quadratics and triangular numbers. Consequent upon this performance, this month's prizewinner is Gareth Suggett of 31 Harrow Road, Worthing, Sussex BN11 4RB. Readers may be sufficiently curious to request copies of Gareth's theory and coding with a view to accelerating his algorithm and reproducing in full the results of Shen Lin.

Those readers who are still deterred by the mathematical background required for some 'Numbers Count' problems are encouraged to read *Mathematical Puzzling* by Tony Gardiner, Oxford University Press 1987, price £4·95, where a considerable spectrum of puzzles are discussed without the need for either mathematical prerequisites or a computer!

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ABBREVIATIONS

3 V.21 (300 baud)
1275 V.23 (1200/75)
12 V.22 (1200/1200)
24 V.22bis (2400/2400)
3-12 V.21, V.22. V.23
3/1275 V.21, V.23
3-24 V.21, V.22, V.23, V.22bis
v viewdata
s scrolling (not viewdata)
h half duplex
r/b ring back
M MNP error correction
* Fidonet node
Most scrolling systems are 8 bits, no parity, 1 stop bit.
Most viewdata systems are 7 bits, even parity, 1 stop bit

NUMBERS COUNT

Mike Mudge investigates prime residue indices and Artin's Constant

Definition (1) Given a prime number p, the prime period length, L(p), is defined to be the number of digits in the period of the decimal expansion of the reciprocal of p (see 'Numbers Count', *PCW*, November 1985). **Note**: primes 2 and 5 are excluded from this discussion since their reciprocals generate finite and hence non-periodic decimal expensions.

Definition (2) Given a prime number p, the prime residue index, i(p), is defined by the quotient $i(p) = (p-1)/L(p)$.

For example, if p = 11, then 1/p = 0.090909 ... *viz* $0.\dot{0}\dot{9}$, and thus L(p) = 2, i(p) = (11−1)/2 = 5.

If p = 31, then 1/p = $0.\dot{0}32258064516129\dot{}$ and thus L(p) = 15, i(p) = (31−1)/15 = 2.

Problem (A) For prime residue indices 1 to 100 (and beyond) determine the smallest prime, $p_{min.}$, possessing each index (see the example in Fig 1).

Residue indices have been discussed at length in, for example, *Studies in Mathematical Analysis and Related Topics*, Stanford University Press, 1962, pp202-210.

Definition (3) The fraction of all primes (excluding 3 & 5) having residue index i is denoted by A_i. Formally this definition may be written as shown in Fig 2.

Now Professor DH Lehmer has conjectured that for i greater than 1:

$$A_i = \frac{A_1}{i^2} \prod_{q|i} (q^2-1)/(q^2-q-1) \text{ where}$$

(i) A_1, the fraction of all primes (excluding 3 & 5) having prime residue index 1 unity is known as Artin's Constant and has been determined empirically to be approximately 0.3739558;

(ii) π indicates the repeated product (thus $\prod_{q=5,6,7}(q+1) = (5+1)(6+1)(7+1)(=336)$; and

(iii) q|i indicates that the repeated product is to be taken over those factors corresponding to the *prime* divisors of i. For example:

$$A_2 = \frac{A_1}{2^2}(2^2-1)(2^2-2-1) = 3A_1/4$$

$$A_6 = \frac{A_1}{6^2}(2^2-1)/(2^2-2-1)((3^2-1)/(3^2-3-1)) = (A_1/36)(3)(8/5) = 2A_1/15$$

i(p)	1	2	3	4	5
$p_{min.}$	7	3	103	53	11

$$A_i = \begin{cases} \text{Limit as} & \text{Number of primes less than or equal to x with} \\ \text{x tends} & \text{residue index i} \\ \text{to infinity} & \text{Number of primes less than or equal to x} \end{cases}$$

i	A_i	B_i	$1-B_i$	$1/(i+1)$
1	0.3739558			
2	0.2804669	0.6544227	0.3456	0.3333
4	0.0701167	0.7910204	0.2090	0.2000
6	0.0498608	0.8597758	0.1402	0.1429

i	1	2	5	6
Number predicted by Lehmer	3740	2805	189	499
Number counted	3755	2808	194	496

Top to bottom: Figs 1-4

Problem (B) For i = i to 36 (and beyond) tabulate A_i, $B_i = \sum_{j=1}^{i} A_j$, $1 - \sum_{j=1}^{i} A_j$ and compare this final quantity with $1/(i+1)$ (see Fig 3).

Problem (C) Using a convenient number of primes, count how many have a given residue index and compare the result with that predicted by Lehmer. Show statistically how the agreement improves with increasing numbers of primes. The target results based upon the first 10000 primes are shown in Fig 4.

Readers are invited to send their attempts at some, or all, of the above problems to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141 to arrive by 1 November 1988. It would be appreciated if such submissions contained a brief description of the program and a summary of the results obtained in a form suitable for publication in *PCW*.

These submissions will be judged using subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution received.

Please note that submissions can only be returned if a suitable SAE is provided.

Review, February: A Chess Board Problem

This somewhat novel area for a 'Numbers Count' problem generated considerable interest. Readers new to the problem are encouraged to read *Mathematical Puzzling* by Tony Gardiner, Section 26, pp121-124. The solutions for 'Queens' are:

n 5 6 7 8 9 10 11 12 13 14 15 16 17
f(n) 3 4 5 5 5 5 5 6 7 8 9 9 9

and, as an appetizer for further research, k(9) = 14 while k(11) = 21.

This month's prizewinner is Frank Webster of 125 Coniston Grove, Middlesbrough, Cleveland TS5 7DF, who used BBC Basic running on an Electron to investigate this problem.

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Personal Computer World, Sept. 1988

ENDZONE

NUMBERS COUNT

Some primes are rare, but some are *very* rare, as Mike Mudge explains.

Trivial thought: the rarest type of prime number is the *even* prime number! There is only one and its value is 2.

Sierpinski Primes These are of the form $p_S = n^n + 1$ and were first considered in 1958. Only two are known, 5 and 257. It is known that any others have more than 3×10^{10} digits. And there (infinitely) (m)any others?

Wilson Primes These satisfy $(p_W - 1)! \equiv -1 (\mathrm{mod}\ p_W)$; where $n! = 1 \times 2 \times 3 \times 4 \times 5 \ldots xn$ and $a \equiv b (\mathrm{mod}\ c)$ means that $a - b$ is an integer multiple of c.

Only three are known, 5 and 13 having been added to in 1953 (Goldberg) by 563. Keller has searched upto 3×10^6.

Are there (infinitely) (m)any others?

Wieferich Primes These satisfy $2^{p_{WH}-1} \equiv 1 (\mathrm{mod}\ p_{WH}^2)$. Only two are known, 1093 (Meissner, 1913) and 3511 (Beeger, 1922); in 1981 Lehmer searched up to 6×10^9.

Are there (infinitely) (m)any others?

Cullen Primes These are of the form $p_C = n \times 2^n + 1$ and six are known: the corresponding values of n being 1; (Robinson 1958) 141; (Keller 1984) 4713, 5795, 6611 & 18496.

Are there (infinitely) (m)any more?

Sophie Germain Primes These are primes p_{SG} such that $2xp_{SG} + 1$ is also prime.

Note If p is a Sophie Germain Prime then there are no integers x, y, & z, different from zero and not multiples of p such that: $x^p + y^p = z^p$.

Are there infinitely many?

Associated Cullen Primes These are of the form $p_C^* = n \times 2^n - 1$ and six are known: the corresponding values of n being 2,3,6,30,75 & 81.

Are there (infinitely) (m)any more?

Problem Readers are invited to construct computer programs to investigate primes of the general form:

$$p = a \times b^n \pm c$$

Particular emphasis should be placed on c = 1, b = 2 with values of 'a' including 3,5,7,9,11,17,19,21,33,45 & 57. However, $a = k^2$ and $a = k^4$ with b = 2 and c = 1, also b=10 and c = 1 are of interest. **Note** with b = 2, n must be greater than 1, otherwise the well-trodden path of Mersenne Primes will be followed.

Test data $17 \times 2^n + 1$ is prime when n = 3, 15, 27, 51, 147, 243 . . .; $45 \times 2^n - 1$ is prime when n = 1,2,3,4,5,6,8,9,14, 15 . . .

Problem II What is the most efficient algorithm for listing the Sophie Germain Primes referred to above? How do these appear to be distributed within the set of all primes? Does this apparent distribution suggest an infinite number of these (female) primes?

Readers are invited to send their attempts at these problems to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 November 1988. It would be appreciated if such submissions contained a brief description of the program and a summary of the results obtained in a form suitable for publication in *PCW*.

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Review, March

The results on the computation of p(x) are summarised by Paulo Ribenboim, *The Book of Prime Number Records*, Springer-Verlag, 1988, culminating with $p(4 \times 10^{16}) = 1075292778753150$.

This book provides the ultimate source of information, at the present time, for any reader interested in Prime Number results and related topics. Numerous readers computed p(x) up to at least $x = 10^8$ . . . with or without error, but there was little interest in the efficiency of the computation. Perhaps 'dummy costing' should be introduced into personal computing to provide a motivation for efficiency?

Despite Ribenboim's assertation: 'It should be noted tht tables of primes on cards or tape are obsolete, because it is possible to generate all the primes p to any given bound . . . quicker than one can read from card or tape', Russell Lavelle-Langham has offered all primes in order up to 50,000,000 on floppy disk at cost. Further details from 31 Risby House, Barleycorn Way, Limehouse, London E14 8DF.

The nearly pattern involving palindromic squares prompted a number of interesting replies, the most detailed of which was from David Poyner of 106 Waterhouse Moor, Harlow CM18 6BE. David defined an algorithm to derive successive values in the progression and established how the arrival of the impermissible *digit 11* at the eighth stage was responsible for the pattern breakdown. David is the worthy winner of this month's prize.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles; all letters will be answered in due course.

Isolated readers can be put in contact with others sharing the same interests. However, greater efficiency regarding published problems should result from contacting the prizewinner.

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NUMBERS COUNT

Mike Mudge investigates prime number density among simple polynomials.

This article concentrates upon only two questions from what is readily seen to be an immense area for conducting empirical research in number theory. Interested readers are encouraged from the outset to consider more general circumstances when writing programs and analysing their output.

Question 1 When is a quadratic polynomial with integer coefficients prime?

We consider the quadratic polynomial $f(x)=ax^2+bx+c$; where a, b and c are given integers and x takes integer values from 0 to N. $V(f(x),N)$ is defined to be the number of x-values for which the modulus of f(x) is either prime or unity.

Test Case. For the Euler Polynomial $f(x)=x^2+x+41$ it is found that $V(f(x),1000)=581$.

E.Karst, New quadratic forms with high density of primes *Elem d Math* vol 28 1973, pp116-118, found a polynomial $g(x)=ax^2+c$ for which $V(g(x),1000)=598$.

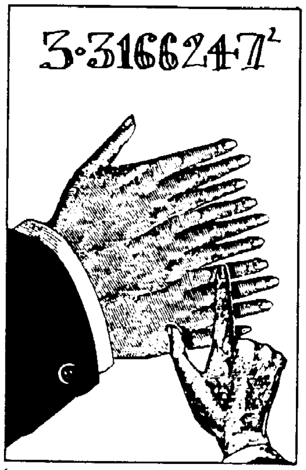
What is it? Can $V(f(x),1000)$ be greater than 598 for quadratic f(x)?

Question 2 For how long can a polynomial of the form x^n+b remain composite (non-prime) where n and b are given positive integers and x takes the values 1,2,3,4...?

Consider the polynomial x^6+1091; this is composite for $x=1,...3905$, (Shanks, 1971).

However there exists a value for b such that x^6+b is composite for $x=1,...7979$. What is it?

Consider the polynomial $x^{12}+4094$; this is composite for $x=1,2,...170624$; however, there exists a value for b such that $x^{12}+b$ is composite for $x=1,2,...616979$. What is it? See



for example KS McCurley, The Smallest Prime Value of x^n+a, *Canadian J Math*, vol 38, 1986, pp925-936, also Polynomials with no small prime values, *Proc Amer Math Soc* vol 97, 1986, pp393-395.

Readers are encouraged to construct programs to evaluate firstly $V(ax^2+bx+c,M)$ for given a,b,c and M, and secondly the length of the initial composite value interval for x^n+b given n and b.

Test data for both programs and suggested targets for an initial search are to be found in the above text.

Readers are further invited to send their attempts at these problems to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141 to arrive by 1 December 1988. It would be appreciated if such submissions contained a brief description of the programs and a summary of the results obtained in a form suitable for publication in *PCW*.

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Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Review, April: Cryptology

This subject area attracted a remarkable response, including a detailed communication from Athens and several from Australia. Many submissions were quite sophisticated and all concerned are to be congratulated upon the interest which they have shown in this topic. It is hoped to produce a related Numbers Count article in the near future; meanwhile here is a selection of interesting points.

Firstly those readers who wish to study codes and cyphers in depth should consider contacting The American Cryptogram Association (ACA) at 12317 Dalewood Drive, Wheaton, Maryland 20902, USA. The Association presently has about sixteen members in the UK.

An interesting program in turbo BASIC suitable for an Amstrad PC1512 has been supplied by MR Barge, whilst the following readers' challenge is due to TK Boyd:

A3 123 B4 135 93 13D BF 13F 81 121 5A 11E 12D 91 B0 9A C0 129 89 67 F9 92 2A F4 E1 11D BC 108 7F 107 69 147 117 66 65 7D E5 108 116 92 9E B7 11A D6 AE 92

It was generated on an Acorn BBC B in Basic II — 'it may matter.' Note: TK Boyd also markets a full-blown 'user-friendly' encryptation package. Details of either of these are available on request.

This month's very worthy prizewinner is Mr Anthony Quas of 635 King's College, Cambridge, CB2 1ST, whose program was written initially in APL 68000 on a SAGE and then transferred to APL*PLUS/PC on a PC, and finally into I-APL/PC.

Anthony will be delighted to discuss his work further with any interested readers, and can readily explain the underlying abstract algebra. A significant contribution of this work is the extension of the scope of the exponentiation cipher to deal with products of distinct primes.

An introduction to many aspects of this work can be obtained from *Cryptography. A primer*, by AG Konheim, Wiley-Interscience, New York 1981, whilst more experienced readers may consult PJ Hoogendoorn, On a Secure Public-key Cryptosystem in Computational Methods in Number Theory by HW Lenstra jr and R Tijdeman, part I pp159-168, *Math Centre Tracts 154*, Amsterdam, 1982.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles; all letters will be answered in due course.

Isolated readers can be put in contact with others sharing the same interests. However, greater efficiency regarding published problems should result from contacting the prizewinner.

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Prime suspect

Mike Mudge explains a persistence property of the positive integers resulting from the addition of their prime factors.

The problem to be investigated this month has been suggested by Paul Cleary, of Mexborough, South Yorkshire.

It is well known that any given positive integer can be uniquely represented as a product of prime factors. Having carried out this factorisation, the resulting factors are added to generate another positive integer and the process is then repeated.

For example, $117780 = 2\times2\times3\times5\times13\times151$; the sum of these factors is $176 = 2\times2\times2\times2\times11$; the sum of these factors is 19, which being a prime number will reduce no further.

Now, if the positive integers from 2 onwards are subjected to the above iterative process it becomes clear that very many reduce to 5, that is, the sum of the two smallest prime numbers 2 and 3.

For example, $148980 = 2\times2\times3\times5\times13\times191$ yielding $216 = 2\times2\times2\times3\times3\times3$, yielding $15 = 5\times3$ and in turn $8 = 2\times2\times2$ finally $6 = 3\times2$ hence 5.

Note: The prime numbers themselves reduce no further under this procedure so are omitted from consideration.

Paul Cleary has listed all the positive integers less than 10001 which reduce to 5 and has noted the occurrence in his list of a number of consecutive sequences (for example, 800, 801, 802, 803, 804) together with a 'sprinkling' of palindromes (such as 444, 484, 959).

Problem I Implement the above iterative procedure and examine in particular the sets of positive integers which reduce to prime numbers other than 5. In passing, it would be valuable to note the distribution of the persistence of the positive integers under this procedure, that is, the number of iterations needed to reach a prime number. This $p(117780) = 2$ while $p(148980) = 5$ from the above examples.

Problem Ia Repeat the above investigation but neglect the multiplicity of the prime factors, that is, add each distinct prime factor once only to obtain the positive integer for use at the next stage.

Problem II Consider an iterative procedure involving the sum of the squares (or indeed any other positive integer power) of the prime factors.

Note Careful consideration must be given to the question of the convergence (or termination) of the procedure for higher powers!

Problem IIa As II, but again neglecting multiplicities.

Readers are invited to send their attempts at some or all of these problems to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 January 1989. It would be appreciated if such submissions contained a brief description of the programs and a summary of the results obtained in a form suitable for publication in *PCW*.

Review, May

Submissions relating to this problem squares of non-consecutive integers giving rise to difference tables having constant second differences, contained a considerable variety of material, ranging from almost random experimentation to very sophisticated algebraic analysis.

However, the prizewinner this month is Robin Merson, of 2 Vine Close, Wrecclesham, Farnham, Surrey GU10 4TE. Robin's submission extends to 15-plus pages of algebraic analysis together with extensive programming of his 'rapidly getting obsolescent' Apple II (indeed the associated Epson 80MX printer 'gave up' part-way through the investigation).

Related Reading

The attention of number theory enthusiasts is drawn to the recent publication of *Elementary Theory of Numbers* by W Sierpinski, editor A Schinzel, from North Holland Mathematical Library. This is the second, revised and enlarged English edition, of 1988. ISBN 0-444-86662-0, 513 pages in hardback only.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future Numbers Count articles. All letters will be answered in due course.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie

Why will 1989 pennies be likely to fetch almost £20?

Prize Puzzle

Not too difficult this month. When Harry began his new job he was told that his weekly wage, which was in excess of £40, would be increased by 99p every pay day. Harry had been paid a total of £407 since he started, and he was soon expecting to break the £60 a week barrier.

What was his starting wage?

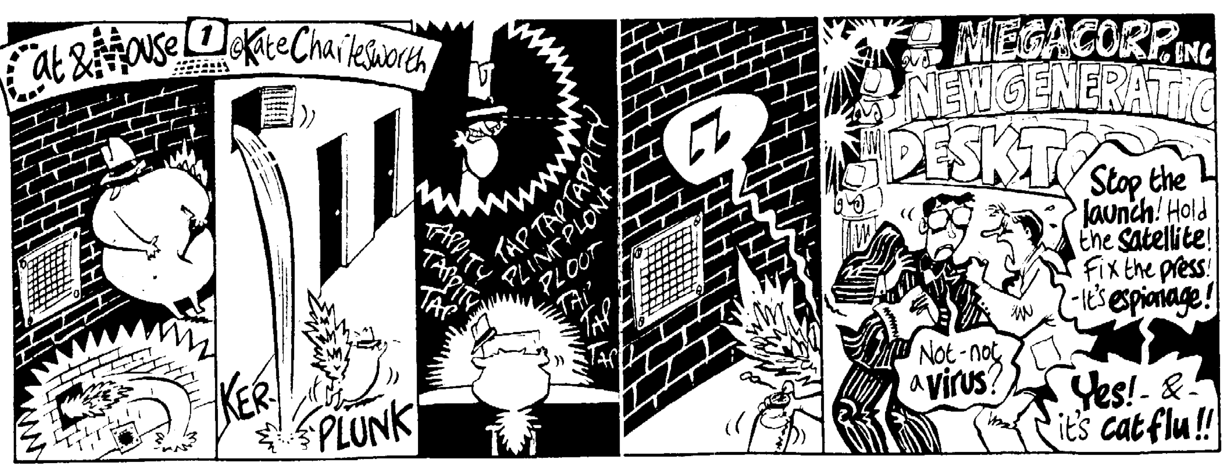
Answers on postcards or backs of envelopes to arrive not later than 30 November 1988. Send your entries to: November Prize Puzzle, Leisure Lines, *PCW* Editorial, VNU House, 32-34 Broadwick Street, London W1A 2HG.

Prize Puzzle August

A reasonable response to this micro-solvable problem — almost 65 entries were received. The winning entry came from Mr DJ Allen of Penn, Bucks, who receives our congratulations. His prize will be on its way shortly.

The winning solution was: 770 combinations which exceed 100.

If you didn't win this time, don't give up — it could be your turn next.



NUMBERS COUNT

Mike Mudge's mathematical mysteries.

Much of the background to this month's problem is to be found in *Elementary Theory of Numbers* by W Sierpinski & A Schinzel, North Holland Mathematics Library, volume 31, 1988: essential reading for everyone interested in Number Theory.

Take three cubes

For the equation $x^3+y^3+z^3=n$, we seek integer solutions for x,y and z; the parameter n takes particular positive integer values.

Case 1 n=2. Here there are infinitely many solutions given by: $x=1+6m^3$, $y=1-6m^3$, $z=-6m^2$ where m is an arbitrary natural number; there are other solutions, however, which are not given by these formulae.

Case 2 n=3. Here there are solutions $(x,y,z)=(1,1,1)$, $(4,4,-5)$ $(4,-5,4)$ and $(-5,4,4)$. Are there any others?

Case 3 If n leaves remainder 4 or 5 when divided by 9 there are *no* solutions. (When n=4,5,13,14,22,23, and so on).

Case 4 n=6. Here there are solutions $(x,y,z)=(-1,-1,2)$, $(-43,-58,65)$ and $(-55,-235,236)$ together with their permutations by symmetry. Are there others?

Case 5 n=30. Nothing is known about this problem.

Higher powers

The equation $x^4+y^4+z^4=t^4$. Nothing is known about integer solutions for x,y,z and t.

The equation $x^4+y^4+z^4+t^4=u^4$ probably has infinitely many solutions in positive integers x,y,z,t and u having no common factor. Thus (30,120,274,315,353) Norrie 1911; there are precisely 81 other solutions with u less than or equal to 20469 — what are they? ... $2^4+2^4+3^4+4^4+4^4=5^4$; $4^4+6^4+8^4+9^4+14^4=15^4$; $1^4+8^4+12^4+32^4+64^4=65^4$.

The equation $(n_1^4-n_2^4)(n_3^4-n_4^4)=m^2$ has solutions $(n_1,n_2,n_3,n_4,m)=(3,2,11,2,975)$ and $(2,1,23,7,2040)$; are there any others?

Readers are invited to send their attempts at some or all of these problems to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, South Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 February 1989. It would be appreciated if such submissions contained a brief description of the programs, details of the hardware used, run times and a summary of results obtained together with suggestions for further investigation in a form suitable for publication.

These submissions will be judged using suitable subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date.

Review, June

Suffice it to say that addition chains was not a very popular topic; but extensive references are to be found in RK Guy's book *Unsolved Problems in Number Theory*, page 63 (or SAE to Mike Mudge). However, an outstanding submission was received from Herr M Meuser of Aloysiusstrasse 13,4047, Dormagen 5, West Germany.

The program written in 8080 assembly language was run on a Bondwell Model 2 with Z80 CPU, 50k of free memory and operating system CP/M. The number of addition chains of length 1 for the final value n denoted by ch(n,1) was computed and L(n) deduced from ch(n,1) as the smallest value of 1 for which ch(n,1) is positive.

Herr Meuser verified the results of Knuth, fig 14 at this stage, also conjecturing that $ch(2+1,2n)=(n!)^2$, $ch(2n,2n-1)=n((n-1)!)^2$ and further that $ch(n,n-1)+ch(n+1,n-1)=ch(n+1,n)$.

Sample output from this very worthy prizewinning entry follows in an attempt to encourage further work.

Length	3	4	5	6
Number of chains	7	36	250	2214

Table 1: Number of all chains of a given length

n	3	4	5	6	7	8	l(n)
4	2	·					(2)
5	2	4					3
6	2	8	12				3
7	0	6	24	36			4
8	1	7	37	108	144		3
9	0	3	29	150	432	576	4
10	0	4	37	218	894	2304	4
11	0	0	19	185	1103	?	5
12	:	3	29	248	1614	?	4
13		0	10	157	1452	?	5
14		0	16	204	1875		5
15		0	4	112	1423		5
16		1	13	173	1910		4
17		0	2	68	1184		5
18		0	7	128	1670		5
19		0	0	37	900		6
20		:	6	106	1495		5
21			0	31	755		6
22			0	48	1058		6
23			0	4	416		6
24			4	68	1067		5
25			0	14	371		6
26			0	24	670		6
27			0	5	267		6
28			:	26	620		6
29				0	152		7
30				12	423		6
31				0	80		7
32				20	435		5
33				2	110		6
34				4	201		6
35				0	51		7
36				12	314		6

Table 2: Number of chains of a given length for a given final value

Value	5	6	7	8	9	10
Number	6	22	66	297	1190	6337

Table 3: Number of chains for a given final value.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or specific, for future articles. All letters will be answered in due course.

LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Merry Xmas to all our readers!

Quickie

Here's a coded message for you to solve. Each letter represents a letter of the original message. The letter 'Q' is unchanged from this original:

'BFS IEZXSA FSG TCIS BFYSB, QIXSZCD BFSA FSG HFXZS ZIZI, WEA QIXZ ZFS TWCCSZ'

Prize Puzzle

This month's puzzle was submitted by Mrs D McClarnon of Swansea who receives our thanks. It's a number crossword for you to mull over while the turkey is cooking.

Please send the completed grid, cut out and stuck on a postcard or the back of a sealed envelope, to: December Prize Puzzle, Leisure Lines, *PCW* Editorial, VNU House, 32–34 Broadwick Street, London W1A 2HG to arrive not later than the end of 1988.

Prize Puzzle, September

A moderate response — about 75 entries, from as far afield as Nigeria, Poland, Greece and many other exotic places (Scotland, Yorkshire, Cleveland . . .).

The problem wasn't too difficult, although it did call for programs which could handle large numbers. The answer was 390,903,804 and the winning entry came from Mr R Levy of Glasgow. Congratulations Mr Levy, your prize is on its way.

Clues Across

1: 1d squared.
4: Twice 11d.
5: 5d — 4a.
8: 4a squared.
9: 9d * 10.
10: 26a + 30d.
12: 22a − 6d.
13: 23a + 4a.
14: Quarter of 30d.
17: 11d * 10.
18: 23a − 6d.
19: Same as 20a.
20: Same as 19a.
21: 17a + 7d.
22: Half 38a.
23: 35d + 4a.
25: Same as 17a.
26: Square Root of 40a.
27: Twice 4a.
28: 21a − 31a.
29: 4a + 13d.
30: Same as 26a.
31: 8a − 39a.
33: Twice 21a.
36: 36d − 14a.
38: Twice 22a.
39: Twice 29a.
40: Twice 24d.
41: Same as 11d.
42: 1a * 4.

Clues Down

1: Half of 2d.
2: Square Root of 42a.
3: 17a squared.
4: 4a * 10.
5: 18a squared.
6: 7d * 3.
7: One third of 6d.
9: Square root of 15d.
11: Square root of 17d.
13: 13a + 4a.
14: 18a * 10.
15: 5d * 14a.
16: 7 * 9a.
17: Half of 36d.
18: Half of 22a.
19: 38d * 6.
21: 4a + 10a.
22: 39a + 11d.
24: 20a + 38d.
25: Same as 10a.
26: One less than 35d.
29: 31a squared.
30: Twice 7d.
32: 1a − 16d.
33: 10a + 2d.
34: 27a * 12.
35: 32d * 3.
36: 4a * 11d.
37: 28a squared.
38: 24d − 19a.

Note:
All answers are whole numbers.
28a = the answer to 28 across.
19d = the answer to 19 down.
* = times.
/ = divided by.