LEISURE LINES

Brainteasers courtesy of JJ Clessa.

Quickie

If you write down every positive 2-digit number (that is, from 10 to 99), which digit will you have written the most number of times?

Prize puzzle

(1) Take a 4-digit palindromic number — that is, one which reads the same from right to left as it does from left to right.
(2) Reverse the digits and add the result to give a new number.
(3) Repeat step 2 with the new number until the result becomes palindromic.

To illustrate, suppose we have the number 3883;

3883 + 3883 = 7766
7766 + 6677 = 14443
14443 + 34441 = 48884

which is palindromic after three cycles only.

Two numbers, however, do not yield palindromic results even after 1000 cycles. What are they?

Answers on postcards, please, or backs of envelopes only, to reach *PCW*, Leisure Lines July 1987, 32-34 Broadwick Street, London W1A 2HG, no later than 31 July 1987.

April prize puzzle

A moderate response — over 100 replies. As usual there was a trace of ambiguity in the problem — does the digit zero follow the digit 9 in the definition of 'consecutive'?

We decided not, since the problem already stated '. . . digits 0–9 are used . . .', which really precludes the ambiguity. Anyway, most entrants who realised the possible ambiguity, sent in the correct solution as well — which was 123341234.

The winning solution came from Scotland — from Mr D Poyner of Charlestown. Congratulations.

NUMBERS COUNT

This month Mike Mudge looks at Cyprian's Last Theorem.

This theorem is due to the Reverend DC Stockford of Downside Abbey, Stratton on the Fosse, Bath; acknowledgement is also due to Mr M Kochanski of 7 Courtfield Gardens, London SW5 0PA who has carried out significant empirical and theoretical studies related to the theorem.

Clearly $3^2+4^2=5^2$, the smallest integer-sided Pythagorean triangle, is familiar to many readers; however, it is less well-known, although equally trivial, that $3^3+4^3+5^3=6^3$.

The geometrical model of a cube with side 6 units, dissected into three smaller cubes with sides 3, 4 and 5 units respectively, is an interesting application of computer graphics. Clearly more than three portions have to be dissected and then some reassembled. What is the smallest number of parts needed?

Cyprian's Last Theorem

$$\sum_{r=1}^{k}(x-1+r)^k=(x+k)^k$$

has no r=1 solutions in positive integers other than x=3 with k=2 or 3.

Note. The notation on the left-hand side is simply shorthand for $x^k+(x+1)^k + (x+2)^k \ldots (x+k-1)^k$ there are k terms.

As an appetiser readers are first invited to find 64 consecutive positive integers, the sum of whose cubes is a perfect cube. It is known that only one such set exists. What about the sum of the n^{th} powers of 64 consecutive integers being an n^{th} power?

What about the sum of the n^{th} powers of k consecutive integers being an n^{th} power?

What about the sum of the n^{th} powers of two integers being an n^{th} power? . . . Fermat's Last Theorem.

Readers are invited to send their thoughts together with complete or partial attempts at the investigations of the above questions to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 October 1987.

It would be appreciated if such submissions contained a brief summary of results obtained in a form suitable for publication in *PCW*. These submissions will be judged using subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date.

Please note that submissions can only be returned if a stamped addressed envelope is provided.

Review: January 1987

This problem involving Bernoulli's Numbers, Euler's Numbers and the connection with Regular Primes can be further studied by reference to *13 Lectures on Fermat's Last Theorem* by Paulo Ribenboim (Springer Verlag 1979). It proved to be a very popular problem among regular contributors but did not appeal to new readers. Why was this so?

The very worthy prizewinner was John B Cook of 34 Joan Crescent, East Burwood, Victoria 3151, Australia. John used a Tandy TRS-80 Model 4P to compute N(X) and D(X), to find irregular primes as defined in the article, and to calculate and factorise E(X), the latter up to X=28.

Test data, together with much other interesting material is to be found in *A Handbook of Integer Sequences* by NJA Sloane (Academic Press 1973).

It must be recorded, however, that Geoff Lockwood of 254 Crystal Palace Road, London SE22 9JH computed Bernoulli Numbers up to the 200^{th} halting then because the computation was taking two hours per number.

Geoff, however, unfortunately did not have time to consider the computation of the Euler Numbers but obtained some rather interesting results comparing true Bernoulli Numbers with the asymptotic formula in Ribenboim's book referred to above.

Mike Mudge welcomes correspondence on any subject within the areas of number theory and other computational mathematics. Particularly welcome are suggestions, either general or particular, for future Numbers Count articles; all letters will be answered in due course.

Isolated readers can be put into contact with others sharing the same interests. However, greater efficiency regarding published problems should result from contacting the prizewinner directly.

NUMBERS COUNT

Mike Mudge delves into the factorisation and other properties of Fermat Numbers, with mathematical requirements being kept to the minimum.

Definition 'Fermat Numbers' are defined by $F_n = 2^{2^n} + 1$. Thus, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, $F_4 = 65537$.

It can be readily verified that these first four Fermat Numbers are prime; indeed, Pierre De Fermat (1601-1665) conjectured that *all* F_n were prime. However, such are the dangers of generalisation based upon empirical evidence, and in 1732 Leonhard Euler found that:

$F_5 = 2^{2^5} + 1 = 2^{32} + 1 = 4294967297$
$= 641 \times 6700417$

In 1880 F Landry proved that:

$F_6 = 2^{2^6} = 2^{64} + 1 = 274177$
$\times 67280421310721$

No prime Fermat Number has been found beyond F_4, so that Fermat's conjecture has not proved to be a very happy one. It is perhaps more probable that although the number of prime F_n is finite, there are others waiting to be discovered (reference the probabalistic argument given as a footnote in *An Introduction to the Theory of Numbers* by GH Hardy and EM Wright).

The existence of prime F_n has an interesting geometrical connection, since Karl Friedrich Gauss proved that a regular polygon having F_n-sides could be inscribed in a circle by Euclidean methods if F_n is prime. (A 65537-sided regular polygon inscribed in a given circle could provide an interesting challenge in computer graphics, particularly if an attempt was made to simulate the Euclidean methods mentioned above!)

Theorem 1 No two Fermat Numbers have a common divisor other than 1.

Theorem 2 If F_n is prime, then the number:

$$Z_n = 3^{2^{2^n - 1}}+1$$

is divisible by F_n.

For example, $F_2 = 17$ is prime, hence we know that $3^{2^3} + 1 = 3^8 = 6561 + 1$ is divisible by 17. In fact, $6562 = 17 \times 386$.

Note The converse of theorem 2 is also true: that is, if F_n is *not* prime, then Z_n is *not* divisible by F_n.

Theorem 3 Any factor of F_n has the form $k \times 2^m + 1$ where $m \geq n + 2$ and k is an odd integer.

For example, the factor 274177 of F_6 cited above is given by $256 \times 1071 + 1$, while the factor 6700417 of F_5 is given by $52347 \times 128 + 1$.

This month's project is to search for factors of Fermat Numbers; it is suggested that Theorem 3 be used, hence two different ways of organising the search are possible.

The description which follows is due to Professor Wilfrid Keller of the University of Hamburg, who has conducted extensive research in this area using both a Telefunken TR 440 computer in TAS assembly language and a Siemens 7.755 with built-in extended precision floating point arithmetic.

How far can *PCW* readers get with this work?

Approach 1 — Trial division For fixed n, look for all k less than some search limit L_n to see if $k \times 2^{n+2} + 1$ divides some F_r, $r \leq n$.

Approach 2 — Tabulation of primes For fixed k, list all primes of the form $k \times 2^m + 1$ for m up to some limit M_k. Then, for each prime, look to see if it divides some F_n where $n \leq m - 2$.

Readers are invited to send their thoughts, together with attempts at this project, to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Staffordshire WV4 5NF, tel: (0902) 892141, to arrive by 1 December 1987.

It would be appreciated if such submissions contained a brief summary of results obtained, in a form suitable for publication in *PCW*. These submissions will be judged using subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date.

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Belated review: September 1986

Regular readers of Numbers Count will be aware that the prize award associated with this problem was deferred (*PCW*, March) due to a lack of response. Interesting correspondence has since been generated, resulting in a very worthy prizewinner: Fred Hartley of 46 Hughes Road, Hayes, Middlesex UB3 3AP. Fred used a BBC Model B and relied very much upon a set of long integer arithmetic routines written in assembler which cope with integers up to 256 bytes in length.

The computing went hand in hand with a careful theoretical analysis, and Fred's third communication concludes: 'I suspect that s(k)/k increases without limit as the number of factors increases, but I have not proved this.' Can any mathematicians help?

I am certain that Fred would welcome enquiries from interested readers regarding the details of his work.

Review: March 1987

This problem was, as expected, extremely popular. It was prompted by the following results quoted in LE Dickson, *History of the Theory of Numbers, Volume 2.*

'Fermat noted that if in (205769, 190281, 78320) we add the area to the square of the sum of the legs, we get a square.

'Frenicle stated that in (17, 144, 145) the sum of the area and the hypotenuse is a square, while the first three right triangles in which the sum of the area and smaller leg is a square, are (3, 4, 5); (16, 30, 34) & (105, 208, 233).

'"Calculator" found three right triangles of equal perimeter and areas in arithmetical progression (18601944, 13951458, 23252430); (18559223, 13999464, 23247145) & (18515584, 14048388, 23241860)'.

While AH Beiler, *Recreations in the Theory of Numbers*, reports:

'Four primitive Pythagorean triangles having a common perimeter have also been found. Only seven such quadruples exist for a perimeter less than 1000000 so they are quite rare. The smallest of these perimeters is 317460 and the triangles are (153868, 9435, 154157); (99660, 86099, 131701); (43660, 133419, 140381) and (13260, 151811, 152389). Can the reader find the other six?'

Unfortunately, the italics in part (v) of the problem together with the lack of the adjective 'primitive' generated a great deal of computer output — at least one complete answer to that part!

After much consideration, this month's prizewinner is Peter Hicks of 9 Carramar Street, Rye, Victoria, 3941, Australia.

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MUSICAL INTERLUDE

TRACKS & PATTERNS

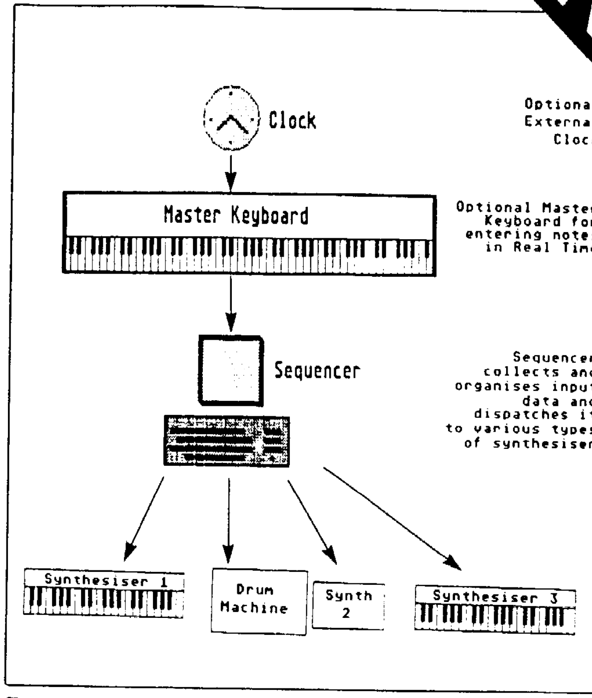
Sequencers often split the computer's memory into different areas or 'tracks' — think of them as separate documents on a word processor, except that the tracks are all audible (or on the screen) at one time. There may be 60 or more tracks available but only one may be altered at any time. These could have totally different things recorded on them, and can be edited, copied or deleted as desired. Furthermore, tracks can usually be sub-divided into patterns, which can be manipulated in similar ways. This system avoids re-entering identical data throughout a song which, for example, has four verses. On most instruments these verses are exactly the same. The verse pattern need only be recorded once for each instrument and then copied and moved around as required.

CRITERIA

When comparing sequencers, check the amount of memory available for note storage, both in terms of RAM and disk, editing facilities, the number of tracks and/or patterns available and the clock resolution (which is as crucial to the quality of recording as the screen resolution is to graphics).

External clocking is vital if a sequencer is to be used in recording studios, as it allows the sequencer to be synchronised with a tape recorder so that different sequences can be recorded on the same piece of tape and still be in time with each other.

Roger Howorth is a freelance computer journalist and sound recording engineer who owns and experiments musically with an Atari ST. If you would like to share your musical experience with Roger or you would like to pass on any interesting snippets, why not write to him care of *PCW*, 32-34 Broadwick Street, London W1A 2HG.



The diagram shows a basic set-up for a sequencing system (all detail has been omitted)

NUMBERS COUNT

Magic squares, sums and cubes — no, it's not a combination of Rubik's Cube and Paul Daniels, but this month's Numbers Count column from our own mathematical marvel, Mike Mudge.

Definition A 'magic square' of order n is a table of n^2 natural numbers written in n rows and n columns such that the sum of the numbers of each row, the sum of the numbers of each column, and the sum of the numbers in the two principal diagonals are all equal.

Albrecht Dürer, a 16th century German artist, made a famous engraving entitled 'Melancholy' which contains the magic square of order 4:

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

in which the sums mentioned above are all equal to 34.

In *Problems in the Theory of Numbers* (Pergamon Press, 1964) W Sierpinski quotes the following magic squares of order 3 & 4 consisting of prime numbers only:

569	59	449
239	359	479
269	659	149

17	317	397	67
307	157	107	227
127	277	257	137
347	47	37	367

The magic sum in the above cases is 1077 and 798 respectively.

Sierpinski further quotes the *Recreational Mathematics* magazine (October 1981, page 28) as displaying a magic square of order 13, consisting of 169 distinct prime numbers.

The conjecture has been advanced that for n greater than 3e (where e is approximately 2.718), there exists infinitely many magic squares formed from n^2 distinct primes.

Readers are invited to write computer programs to construct and display magic squares of a given order using:

(i) natural numbers less than a specified N;
(ii) natural numbers between specified N_1 and N_2; and
(iii) prime numbers only.

An obvious extension of this work would be to magic cubes where, for example, the magic sum may be required in each plane parallel to the faces or in some other carefully defined region.

Readers are invited to express their thoughts on the possible generalisation of the magic square to three (or more!!) dimensions. Submissions should be sent to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Staffordshire WV4 5NF to arrive by 1 January 1988.

All submissions will be judged using subjective criteria, and a prize will be awarded by *PCW* to the 'best' contribution received by the closing date.

It would be appreciated if such submissions contained a brief summary of results obtained, in a form suitable for publication in *PCW*.

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(April's review on 'W-sequences' will appear in next month's issue.)

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