MICROCHESS

27	e4–e5	Rf8xa8
28	Qc3–c6	Ra8–b8
29	Kg1–f1	Bh3–f5
30	Qc6xc7	Rb8–c8
31	Qc7xd6	Qg5–d2

White has picked up two pawns for the piece, but this is killing.

31	Re1–b1	Bf5xd3+

0–1 (White resigns)

NUMBERS COUNT

Mike Mudge deals with PAPs (Primes in Arithmetic Progression), and presents the winning solution to Euler's Constant problem.

Definitions (i) A *prime number* is a positive integer which is only exactly divisible by itself and unity (one): for example, 2, 3, 5, 7, 11, 13, 17, 19, 23, . . .

(ii) A *palindromic number* is a number which reads the same backwards as forwards: for example, 1234321, 78000087.

(iii) An *arithmetic progression* is a sequence of numbers, each member differing from the previous one by the same constant quantity: for example, 7, 10, 13, 16, . . . or, in general, a, a+d, a + 2d, a + 3d, . . . Weask the question: how long can an arithmetic progression be, which consists only of prime numbers (a PAP)?

It is conjectured that a PAP can be as long as we wish. The truth of this conjecture would readily follow from an improvement to a theorem of Endre Szemeredi (see *Acta Math Acad Sci Hungar*, vol 20 1969, pp 89–104).

Sierpinski defines g(x) to be the maximum number of terms in a PAP not greater than x. The least x, 1(x), can then be regarded as a function of g(x) yielding the following table:

g(x) 2 3 4 5 6 . . .
1(x) 3 7 23 29 157 . . .

The first column refers to the PAP 2, 3, while the fifth column refers to the PAP 7, 37, 67, 97, 127, 157.

It has been conjectured that there are arbitrarily long PAPs of: (a) consecutive primes such as 251, 257, 263, 269, and 1741, 1747, 1753, 1759; and (b) palindromic primes such as 13931, 14741, 15551 and 16361.

Paul Erdös has broadened the problem by conjecturing that if (a_i) is any infinite sequence of integers for which the sum of the reciprocals is divergent, then the sequence contains arbitrarily long arithmetic progressions. He offers a prize of $3000 for a proof or disproof of this conjecture.

Problem A List all the PAPs of a given length contained within a given table of prime numbers.

Tabulate separately those consisting of: (a) consecutive primes; and (b) palindromic primes.

Problem B Extend Sierpinski's table of 1(x) as a function of g(x), printing the PAP to which each entry corresponds.

Problem C Investigate the Erdös conjecture for various sequences (a_i) whose reciprocals have a divergent sum.

Readers are invited to submit their attempts at some (or all) of the above problems to: Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Wolverhampton, Staffordshire WV4 5NF. Tel: (0902) 892141.

Submissions, which must reach me by 1 September 1986, will be judged using suitably vague criteria, and a prize will be awarded to the 'best' contribution received by the closing date.

Please note that submissions can only be returned if a suitable stamped, addressed envelope is provided.

Expanded revues of previous problems, together with, subject to the approval of the contributor, copies of detailed programs from the winning entry, may also be requested. In the interests of efficiency, interested readers are urged to contact the prize-winner directly.

The writer (Mike Mudge) welcomes correspondence on any subject within the areas of number theory and other computationally related mathematics, and will endeavour to reply to all letters after sufficient time has elapsed!

December review

Euler's Constant, defined as the limit as n, tends to infinity of $(1/1 + 1/2 + 1/3 + \ldots + 1/n) - \log_e n = 0.57721566490153286060651209 \ldots$ (see for this, and many other interesting numbers, *A Handbook of Integer Sequences* by NJA Sloane, Academic Press 1973).

The conjecture that $4/n = 1/x + 1/y + 1/z$ could be solved in positive integers for all n greater than 1, has been verified for n less than or equal to 10^8 by Nicola Franceshine; the corresponding result for 5/n is explored up to n = 1057438801 by Stewart, who covers all n *not* of the form 278460k + 1.

There are many other problems — for example: 'What is known about sets of unequal, odd integers whose sum is unity, such as 3, 5, 7, 9, 15, 21, 27, 35, 63, 105, 135?' The interested reader is referred to *Diophantine Equations* by LJ Mordell, Academic Press 1969, and also to *Unsolved Problems in Number Theory* by RK Guy, Springer 1981.

The prize-winner this month is Henry Ibstedt of 4 Rue Gramme, 75015 Paris, who, in addition to suggesting that Folkman's Number is probably bigger than Skewes' Number, used his IBM PC with 256k RAM in Basic to discover 14 140-digit equal denominators in the sums S_{317} to S_{337}, and extended the search to S_{457}. No equal numerators exist between S_1 and S_{457}.

Henry implemented two methods of representing integers with different terms from the harmonic series, expressing 5 as the sum of 1920 such terms and 6 as the sum of 1658880 such terms.

Many other results and investigations were detailed in an extremely well-documented submission, resulting in a most worthy prize-winner.

Well done, Henry.

LEISURE LINES

Brain-teasers courtesy of JJ Clessa.

No answers — no prizes. Which number, when multiplied by three, gives the same result as if it were added to 20?

Prize puzzle

The four-digit number 4151 has the property that the number is equal to the sum of the fifth powers of its digits — $4151 = 4^5 + 1^5 + 5^5 + 1^5$.

MICROCHESS

being better, and I agree.

12	g2xf3	Qd8–a5
13	0–0–0	Ra8–c8
14	Rh1–g1	

Hitech spent most of its time examining Kc1–b1, but then when it got down to 7 or 8-ply it switched to the text move which is very much stronger.

14	...	Rf8–e8
15	Be3–h6	g7–g6
16	Bh6–g5	Qa5–c5
17	Qd2–f4	Nf6–h5
18	Qf4–h4	f7–f6

18 ... Be7xg5 19 Rg1xg5 — this move also gives White a tremendous attack.

19	Bg5–e3	Qc5–a5
20	Be2–b5!	

A pretty example of 'interference' — blocking the fifth rank and interfering with the black queen's defence of the knight.

20	...	Bd7xb5
21	Qh4xh5	g6–g5
22	Be3xg5!	f6xg5
23	Rg1xg5+	
23	...	Kg8–h8

Or 23 ... Be7xg5 24 Qh5xg5+ Kg8–f7 (24 ... Kg8–h8 25 Qg5–f6+ and mate next move) 25 Nc3xb5, threating to win everything with Nb5xd6+, is impossible for Black; for example, 25 ... e6–e5 26 Nb5xd6+ Kf7–e6 27 Qg5–f5+ Ke6–e7 28 Qf5–f7+ Ke7–d8 29 Nd6xb7 mate.

24	Rd1–gl!	1–0 (Black resigns)

There is no adequate defence against the threat of 25 Qh5xh7+ followed by 26 Rg5—h5 mate, since 24 ... Be7×g5 25 Qh5×g5 Rc8–c7 26 Qg5–f6+ Rc7–g7 27 Qf6×g7 is also mate.

NUMBERS COUNT

Mike Mudge tackles divisor functions first posed by M Rumsey.

Definition of the Divisor Function: s(n) Given any positive integer, n, s(n) is defined to be the sum of all of the positive integers which divide exactly (no remainder) into n.

For example, s(98) = 1 + 2 + 7 + 14 + 49 + 98 = 171

s(p) = p + 1 where p is any prime number, which by definition is only divisible by itself and one.

In *Eureka*, volume 26, page 12, 1963, M Rumsey asked for solutions of the equation s(q) + s(r) = s(q + r) ... (i)

We now present a survey of some results relating to this equation.

Result A if q + r is prime, the only solution of (i) is: s(1) + s(2) = (s)3, that is, 1 + 3 = 4.

Result B If q + r = p^2, where p is a prime, then q is prime and r = $2^n k^2$, where n and k are odd integers (or conversely, since (i) is symmetrical in q and r).

The case k = 1 leads to solutions when p = 2^n — 1 (that is, a Mersenne Prime — see for instance *A Concise Introduction to the Theory of Numbers* by Alan Baker, CUP 1986, for a detailed discussion of these particular primes provided that q = $p^2 - 2^n$ is also prime. Such solutions occur for n = 2, 3, 5, 7, 13 and 19. Among the values of n for which the question is, to the best of the author's knowledge, still open are 31, 61, 89, 107, 127, 607, 1279, 4253, 9941 and 11213.

There are no solutions to (i) under result B if k contains a factor which leaves remainder 3 when divided by 14.

The case k = 5 has been shown to yield no solutions except possibly when n = 189, 249, 501, 509, 521, 573, 585, 605, 621, 809, 845, 861, 873, 969 ...

The case k = 7 yields for:n = 1 the solution: s(5231) + s(98) = s(5329) — that is, 5232 + 171 = 5403.

n = 2 the solution:
s(213977) + s(392) = s(214369) — that is, 213978 + 855 = 214833 more easily displayed as s(213977) + $s(2^3.7^2) = s(463^2)$, the next values of n in doubt being 31, 33, 103, 115, 121, 123, 159, 169, 225, 255 ...

The case k = 11 yields for:n=1 the solution:
$s(24407) + s(2.11^2) = s(157^2)$
n = 13 the solution;
$s(1410646926617) + s(2^{13}.11^2) = s(1187707^2)$, the next values of n which are in doubt being 21, 45, 57, 67, 141, 145, 153, 163, 177, 193 ...

The case k = 13 has no known solutions. However, those values of n which are still in doubt commence 53, 55, 79, 91, 149, 163, 175, 187, 229, 277 ...

Other known solutions under result B include:
$s(155015849) + s(2^5.19^2) = s(12451^2)$
$s(1193399) + s(2.5^4) = s(1093^2)$
$s(229405235369) + s(2^9.5^4) = s(478963^2)$
$s(2676857975009) + s(2^9.7^4) = s(1636111^2)$

For n = 1 and k, prime solutions are known for k = 53, 137, 193 & 277, while for n = 3 with k = 313 & 421; also, for n = 5 with k = 97, 107, 131, 149 & 257 yield solutions.

Result C If g + r = p^3, where p is a prime, the solutions known to the author are s(2) = s(6) = s(8) – that is, (1+2) = (1+2+3+6) = (1+2+4+8); also $s(11638687) + s(2^2.13.1123) = s(2227^3)$.

Readers are invited to write a program to evaluate the divisor function: s(n), ideally where n is either input as a general length integer or in terms of its prime factors; investigate solutions of Rumsey's equation (i) above, recovering some (or all!) of the given results, hopefully with some new ones; investigate a somewhat similar equation due to Leo Moser:

m s(m) = n s(n) ... (ii) where m and n are two unequal positive integers.

Note that m = 12 and n = 14 is a solution which in turn leads to an infinity of further solutions m = 12q and n = 14q where q and 42 have no common factor.

Attempts at the above may be submitted to: Mike Mudge, 'Square Acre', Stourbridge Road, Penn, nr Wolverhampton, Staffordshire WV4 5NF, tel: (0902) 892141 by 1 December 1986.

It would be appreciated if such submissions could contain a brief summary of results obtained and thoughts relating to the problem, in a form suitable for future publication in *PCW*. These submissions will be judged using suitably vague criteria, and a prize will be awarded to the 'best' contribution received by the closing date.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Expanded reviews of previous problems, together with, subject to the approval of the contributor, copies of detailed programs from the winning entry may also be requested. In the interests of efficiency, readers are encouraged to contact the prize-winner directly.

Mike Mudge welcomes correspondence on any subject within the

NUMBERS COUNT

areas of number theory and computationally-related mathematics, particularly containing suggested subject areas for future Numbers Count articles, and will endeavour to reply to all letters.

Markoff Numbers

This problem (*PCW*, March) proved inexplicably popular; an extreme solution involving a triple with elements of 95, 59 and 36 digits respectively was produced by an Apricot F1e in South East London using Fortran.

A Tandy Model 4P in Sydney, Australia yielded an ordered list of Markoff numbers up to 99999999.

Several regular contributors gave this problem a lot of attention, having found it to be 'fascinating', 'challenging' and readily amenable to programming in Basic or indeed any other high-level language.

This month's winner, after considerable thought, is John Scholes, of 79 Dryden Court, Renfrew Road, London SE11 4NH who used MIX/C ('a cheap C compiler') on an IBM PC. John follows a theoretical study with full program listings and a print-out of all Markoff Triples less than 10^{12}, there being 152 of them. His approach to the 'related Diophantine equation' yielded only eight solutions in an 18-hour search, the largest unknown being 59. This was improved upon by several other contributors; indeed, one run of over 135 hours yielded two new solutions, (3,3,4,6,42,87) and (2,13,39,97,99).

There is still scope for an efficient search algorithm:

$$5(p^2+q^2+r^2+s^2+t^2)^2 - 7(p^4+q^4+r^4+s^4+t^4) = 90pqrst.$$

LEISURE LINES

Brain-teasers courtesy of JJ Clessa.

My wife went shopping last Saturday and bought three items from successive shops. At each shop, the item she bought cost threequarters of the money she had in her purse at the time. She came home with 10p only. How much did she have in her purse at the outset?

Prize puzzle

This will get the micros whirring.

To multiply 77 by 23, you put the single digit '1' before and after the answer 1771.

To multiply 91 by 32, you put the single digit '2' before and after the 91 to give the answer 2912.

To multiply 411 by 83, you put the single digit '3' before and after the 11 to give the answer 34113.

What is the smallest number that you could multiply by 29 using this method, and what would be the single digit which you would use?

Answers on postcards or backs of envelopes only, please, to reach *PCW*, Leisure Lines Prize Puzzle September, 32–34 Broadwick Street, London W1A 2HE, no later than 30 September 1986.

June prize puzzle

Almost 200 entries this month — denoting a rather easy problem. Quite a cosmopolitan bunch, though, including entries from France, Belgium, Portugal, Italy, Norway, Sweden, Holland, Greece, Zambia, Malta and Pakistan, plus a letter from Switzerland (which we accepted since the writer said that the Lausanne Post Office wouldn't let him send a card!)

We were rather fascinated by a card from Mr Wesencraft of London who claimed that he had a 'truly wonderful proof' that the sum of the first four digits of any answer must be 10 but, unfortunately, his postcard was too small to contain it!

We certainly don't know of this proof but it sounds plausible, and certainly holds for the three answers we have which are:

54748 (the smallest)
92727
93084

The winning entry, drawn at random, came from Brian & Gaynor Steer of Slough.

Congratulations both of you — your prize (one only) is on its way. Meanwhile, to all the also-rans — keep puzzling.

ACC NEWS

A look at computer clubs up and down the country, with Rupert Steele.

Even older than the personal computer itself is the hobby computing movement; indeed, the Amateur Computer Club first met in 1972, and early discussions concerned the ACC's own design of central processor, the 'weeny bitter'. This was a 2-bit machine assembled laboriously from huge numbers of discrete logic chips. (If anybody out there actually has one of these still working, I'd be very glad to hear from them.) Since then, of course, the technology has taken off, and with it the hobby movement.

As well as over three hundred 'public' clubs nationwide, many schools and workplaces have their computer clubs.

The Association of Computer Clubs grew out of this mass of clubs, as a forum for them to get together to arrange space at exhibitions and, more recently, to participate in cheaper insurance schemes. The Association also makes available a list of people who are prepared to give talks at computer clubs. For more details of the ACC's services, see the end of this column.

Many computer clubs close down over August when many of the members are away, and there is little point in meeting. So, by September, everybody is ready to go with the Autumn programme. One such club is OPeCC, the Oxford Personal Computer Club. This is a pretty old club: it was formed when the original OPEC was still a force to be reckoned with. The club meets on the first and third Wednesday of each month at 7.30pm in the Donnington Community Centre, Townsend Square, Oxford. The programme kicks off on 17 September with the Annual General Meeting. Further information is available from N Goodwin of 57 Catherine Street, Oxford, tel: (0865) 244914 or F Cameron of 12 Norreys Avenue, tel: (0865) 240058.

Further south, I have had a note from the Brighton, Hove and District Computer Club which has a new secretary, George Shears, whose address is 19 Beach Green, Shoreham-by-Sea, Sussex BN4 5YG,