Personal Computer World, Numbers Count

- July 1985, p. 224. Eulers "Numeri Idonei"

  Tal d, hvor $d = ab$ og der findes et n, der entydigt (uniquely) kan skrives $n = ax^2 + by^2$ hvor $(ax, by) = 1$.
  Så er $n = p$, $n = 2p$ eller $n = 2^k$ så ulige $n \in P$.
  $d = 1, 2, 3, 4, 5 \ldots 10, 12, 13, 15, 16, 18, 21, \ldots 1320, 1365, 1848$; alt 65 tal.
  Ex: $d = 57 = 3 \cdot 19$ giver $19 \cdot 8^2 + 3 \cdot 577^2$
  $d = 1848 = 1 \cdot 1848$ giver $197^2 + 1848 \cdot 100^2$
  Er der flere end de 65?

- August 1985 p. 226 Ciffermønstre i a^n. Palindromiske produkter som kvadrater.

September 1985 p. 226 Exponentiel Diofantiske Ligning:
$w^a = x^b y^c + w^d x^e y^f \ldots$

Oktober 1985 p 222. Kodebrekker

% kopi! November 1985 p. 222 Periodelængde af $\frac{1}{p}$

December 1985 p. 229 Sums of reciprocals and X

Personal Computer World, Jan. 1985

NUMBERS

Mathematical mind-benders from Mike Mudge

The notion of a congruent number has been familiar to some mathematicians for at least a thousand years. The defining algorithm leading to the construction of such numbers is easily illustrated by a simple example:

(i) Take three squares, $a^2<b^2<c^2$, which have a common difference, D (we say that they are in arithmetic progression): for example, $a^2=1^2=1$, $b^2=5^2=25$, $c^2=7^2=49$, here D=49−25=25−1=24.

(ii) Take the common difference, D, and write it in the form Nd^2, where N is square free: for example, $D=24=2^2\times6$.

(iii) N is then a *congruent number* (6 is such a number).

The congruent numbers 5, 6, 14, 15, 21, 30, 34, 65, 70, 110, 154, 190, 210, 221, 231, 286, 330, 390, 429, and 546, together with 10 more less than a thousand, were given on an Arab manuscript c900AD.

However, it was left to L Bastien in 1915 to establish the congruent number 101, for which the smallest associated integers are:

a=16281243707272699996961,
b=20152424629497600019 61
c=23391484353062250069 61,
d=11817143185277945190 0

Algebraic formalism We are solving $b^2-a^2=c^2-b^2=Nd^2$ and the problem is to discover which values of N are permissible.

Since $2b^2=c^2+a^2$ then $(2b)^2=(c+a)^2+(c-a)^2$, now writing $Nd^2=uv(u+v)(u-v)$ where u is even, v is odd and u and v have no common factor provides the starting point for much of the computation and theoretical study that has taken place. A summary of known results including 198 congruent and 135 non-congruent numbers less than a thousand is given by R Alter and T B Curtz (*Math Comp* vol 28, 1974, pp303).

It is now appropriate to acknowledge the assistance and advice of Robin Merson, one of this column's regular readers. He is responsible for the following new approach and also suggested the inclusion of this topic in the Numbers column.

We give the detailed algebra for odd N; the word of encouragement being that following through this analysis on your micro should readily yield the non-congruent number 105 which is not well known in the current literature: Set $u=4n_1p^2$, $v=n_2q^2$, $u+v=n_3r^2$, $u-v=n_4es^2$, where $n_1n_2n_3n_4=N$ and $e=+1$ or $e=-1$.

So, we have the four equations $4n_1p^2+n_2q^2=n_3r^2$, $4n_1p^2-n_2q^2=n_4es^2$, $8n_1p^2=n_3r^2+n_4es^2$, $2n_2q^2=n_3r^2-n_4es^2$.

Taking congruences modulo 8, it is seen that n_2-n_3 is divisible by 4, and n_3+en_4 is divisible by 8. These are linear restrictions. Taking congruences modulo each n in turn, we obtain quadratic restrictions: for example, n_2n_3 is a quadratic residue of n_1 by which we mean that there is a value of x for which $x^2-n_2n_3$ is divisible by n_1. There are twelve such restrictions which are not completely independent.

Readers are invited to submit a program or programs to determine congruent and/or non-congruent numbers. They may reasonably restrict the search to numbers less than a thousand, although this is not obligatory.

Try the effect of making two or three of u, v, u+v, u−v, perfect squares; this is equivalent to having two or three of n_1, n_2, n_3, n_4 unity. To obtain some non-congruent numbers take a square free N, factorise it, allocate its factors in all possible ways to n_1, n_2, n_3, n_4, testing each allocation for the restrictions. If all ways fail, then N is non-congruent.

Submissions should include program listings, hardware description, run times and output. These will be judged for accuracy, originality and efficiency (not necessarily in that order) and a prize will be awarded to the 'best' entry received by 1 April 1985.

Please address entries to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, near Wolverhampton, Staffs WV4 5NF. Tel: (0902) 892141.

Tree-like structures, August 1984

The 3× + 1 Problem, also known as Collatz', Kakutani's, Syracuse and Ulam's Problem or Hasse's Algorithm attracted considerable response. Volume one, number eight, June 1984 of *Quarch* (Archimedeans, Cambridge University Mathematical Society Newsletter) contains a detailed report on the state of the art; appropriate since this problem first prompted the newsletter in April 1980.

J C Lagarias, *The 3× + 1 problem and its generalisations*, American Mathematical Monthly, 1984/5 surveys the large amount of work done in this field. Responses from *PCW* readers included reference to R E Crandall's *Math Comp*, Vol 32, where the sequence $a_{n+1}=\frac{1}{2}a_n$, a_n even and $a_{n+1}=da_n+1$, a_n odd is discussed. For d=1 this is trivial, if d=5,181 or 1093 the recurrence does *not* reduce to a cycle involving 1 for all a_0. For d=7, a_0=3 the behaviour is unknown.

Dr D Fisher supplied a one-line statement of the problem:

$$x_{n+1}=\tfrac{1}{2}\left(x_n 3^{(1-\cos\pi x_n)/2}+(1-\cos\pi x_n)/2\right)$$

as a special case of a 'chaotic iteration', $x_{n+1}=Lx_n(1-x_n)$. No attempt was made to use any graphics to display tree-like structures; this topic will be returned to in a later problem.

This month's worthy prizewinner is Fred Salt of 'The Paddock', Flanders Road, Llantwit Major, South Glamorgan CF6 9RL. The work was carried out in Apple Pascal on a U200 (Apple compatible) computer with 48k RAM + a 16k language card; the results were displayed using an Epson MX80 FT III printer. A suitable prize will be sent to 'The Paddock'. Further enquiries relating to this work should be directed either to Fred Salt or myself.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Expanded reviews of previous problems together with, subject to the approval of the contributor, copies of detailed programs from the prize-winning entry may also be requested.

END

LEISURE LINES

Brain-teasers courtesy of J J Clessa

Quickie

Put two pennies on the table. If you keep one fixed and roll the other around its edge, always touching, how many revolutions will it make? One, you might think? Try it and see.

Prize Puzzle

I bought a book the other day — it was an exciting mystery story. But I discovered, after I was well into it, that a whole section of pages was missing. I calculated that the total of the page numbers on the missing sheets was 2567. What were the missing pages?

Answers, on postcards only, to: *PCW* Prize Puzzle, January 1985, Leisure Lines, 62 Oxford Street, London W1. Entries to arrive not later than 31 January 1985.

September Prize Puzzle

Over 200 entries were received for this not too difficult logic problem. In fact, the most difficult bit for a Greek entrant was knowing what a 'postcard' was so that he could send in his (correct) quotient of 4.

Incidentally, as several of you observed, if you add or remove 9's from the middle of this number the property still remains.

The winning entry comes from Mr. Claes Malcolm from Sweden. Congratulations Claes (or is it Malcolm?), your prize is on its way.

NUMBERS

Problems with primes from Mike Mudge.

Definitions A Prime number is a positive whole number which is exactly divisible by itself and unity only. Thus the sequence of primes begins 2, 3, 5, 7, 11, 13, 17, 19, . . .

A truncatable prime number is a prime which yields a sequence of primes when successive digits are removed: always from the left (for a left-truncatable prime), always from the right (for a right-truncatable prime), or simultaneously from the left and right (for a shrinking prime). For example: 629137 is left truncatable since it is prime and so are 29137, 9137, 137, 37, & 7. 939133 is right-truncatable since it is prime and so are 39133, 9133, 133, 33, & 3.

The State of the Art Angell IO and Godwin HJ *1977 Mathematics of Computation*, vol 31 page 265, have tabulated, to base ten, the largest left-truncatable prime, L_a, with base a between 3 and 11 inclusive also the largest right-truncatable prime, R_a, with base a between 3 and 15 inclusive.

Keith Devlin in *The Guardian* (8/11/84) broadened the problem, base 10, by admitting 1 as a prime. He stated that there are 147 R_{10} the largest being 1979339339. (This reducing to 83 with largest 73939133 if 1 is excluded.)

Further Devlin counted 403 L_{10} less than 10^4 (this reducing to 308 with the exclusion of 1) together with a total of 24 shrinking-primes (reducing to 9 with the exclusion of 1).

The problems set are:

(i) Reproduce the above results.

(ii) Extend the range of a quoted by Angell & Godwin for L_a & R_a.

(iii) Consider shrinking-primes, S_a, in bases different from 10.

(iv) Investigate what happens if a given prime is to be successively truncated by the removal of primes from either or both ends.

Readers are invited to submit their program listings, together with hardware descriptions, run times, any comments and of course the output relating to some (or all) of the above problems. These will be judged for accuracy, originality and efficiency (not necessarily in that order), and a prize will be awarded to the 'best' entry received by 1 June 1985.

Please address all correspondence to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Nr Wolverhampton, Staffordshire WV4 5NF. Tel 0902 892141.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Expanded reviews of previous problems together with, subject to the approval of the contributor, copies of detailed programmes from the prize winning entry may also be requested.

Prize winner

(Factorial n) + 1 is prime for the following n less than 231: 1,2,3,11, 27,37,41,73,77,116, & 154.

(Primorial p) + 1 is prime for the following p less than 1031: 2,3,5,7,11, 31,379,1019, & 1021.

This month's prizewinner is Gareth Suggett, Chichester, Sussex. Readers should be encouraged by the incompleteness of the winning submission. Gareth has put all of the primes up to 65063 on a data file and implemented some generalised arithmetic routines in Basic on his BBC Micro, in spite of recurring hardware problems. Primorials up to 101 digits and factorials up to 72 digits have been listed. However, the tests for N.T.P. are incomplete being conditional on extending the data file and efficiently coupling it to the remaining programme in a factorisation routine.

Two related areas remaining for investigation are:

a) Define $A_n = n! - (n-1)! + (n-2)! \ldots -(-1)^n 1!$ A_n is prime for $n = 3,4,5,6,7,8,10,15,19$ at least; whilst $n = 27$ yields the first A_n with a square factor. When are the A_n N.T.P.?

b) The left factorial function is defined by: $!n = 0! + 1! + 2! + 3! + \ldots + (n-1)!$ When is $!n$ prime or N.T.P.?

MICROCHESS

Kevin O'Connell bets on Chaos in the North American Computer chess championship.

The game which follows was played in the last round of the 15th North American Computer Championship, held in San Francisco last October.

The game is proof of grandmother's old saying that one should never bet on a proposition.

White: Chaos. Black: Phoenix. Benoni Defence

1	d2–d4	c7–c5
2	d4–d5	e7–e5
3	e2–e4	d7–d6
4	c2–c4	g7–g6
5	Nb1–c3	Bf8–g7
6	Bf1–d3	Ng8–e7
7	Ng1–e2	O–O
8	Bc1–d2	

(A typical position out of the Old Benoni Defence, which shows the great progress made by programs in the last few years. It is very important here to retain freedom of movement for the f-pawns and both programs seem to understand this.)

8	. . .	f7–f5
9	f2–f3	Nb8–a6
10	Bd2–g5	Na6–b4
11	Bd3–b1	h7–h6
12	Bg5–h4	g6–g5
13	Bh4–f2	f5xe4
14	Bblxe4	

(Having e4 for his pieces promises White some advantage.)

14	. . .	Bc8–f5
15	O–O	Qd8–d7
16	a2–a3	Nb4–a6
17	Qd1–b3	Na6–c7
18	Be4xf5	

(The start of an interesting but very risky plan. This makes the c6 square available to White's queen. However, the net result of the whole manoeuvre is merely a very weak white pawn on c6.)

18	. . .	Ne7xf5
19	Qb3xb7	Rf8–b8
20	Qb7–c6	Qd7xc6
21	d5xc6	Rb8xb2
22	Ra1–b1	Ra8–b8
23	Ne2–g3	Rb2xb1
24	Rf1xb1	Rb8xb1+
25	Nc3xb1	Nf5–e7
26	Ng3–e4	Nc7–e8
27	Ne4xd6	

(A horrible decision to have to make, but against anything else Black simply removes the pawn on c6 and then builds up his central power-house.)

27	. . .	Ne8xd6
28	Bf2xc5	Ne7–c8
29	Nb1–d2	Kg8–f7