GOOD HOUSEKEEPING

fiddling the system. In effect the program only allows one source of funds (the bank account). The upshot of all this is that if you use this package, you will not be able to reconcile the accounts produced by the program back to your bank statement, or your Access statement, or any other statement.

Another point to bear in mind generally when you are looking at this kind of very detailed system is that you will need lots of self control to log everything you spend. But if you don't do this it will make a nonsense of the whole system.

It is a pity this program has such basic flaws because it is very well presented and works very fast (much faster than many business programs I've seen). It also has the ability to do some analysis on your expenditure which, had it been executed correctly, has the potential to be very useful. Another potentially useful feature is that the program has been designed to work with both a ZX printer and any Centronics compatible printer using the Kempston interface.

Conclusions

So what is the upshot of all this? I think more and more people are looking for something useful to do with their home computers. However, at the moment, I think, this smacks of 'I've got a home computer so I might as well buy the package to give it something useful to do'. This is the reverse of what should happen. You ought to be saying 'I have a problem. Can a computer help?'.

Home accounts is certainly an area in which a home computer could be useful. However in all honesty I can't recommend either of the programs I have tested here. The OCP Finance Manager is full of good ideas but is let down by its execution. The Diamondsoft Home Accounts works, but it operates at such a simple level that I doubt its usefulness. An ideal system would allow analysis of expenditure and some budgeting but would remain easy and quick to use.

If you feel you would benefit from having your home accounts done by your home computer it may be that your best bet is to buy a simple spreadsheet such as Vu-File, and use that.

END

NUMBERS COUNT

FRACTIONAL APPROXIMATION TO PRIME NUMBERS

New readers start here. The topics dealt with in this column attempt to reach the frontiers of knowledge in number theory with the minimal background information. The problems posed therefore have no complete solution known to the author, and readers are encouraged to submit their attempts at solution, however incomplete they may seem.

A Prime Number is defined to be a positive integer, greater than unity, which is divisible only by itself and unity. Thus the sequence of prime numbers commences 2,3,5,7,11,13,17,19,23,29 . . .

The Modulus of a number, x, written $|x|$, is identical with the Basic function ABS (x) and is defined to be the argument, x, made positive. $|x|=x$, if $x\geq 0$, else $|x|=-x$.

Readers will be familiar with ideas of (i) approximating a real number by a rational number (or fraction), and (ii) approximating a rational number in turn by an integer (or whole number); this latter being the process of rounding off to zero decimal places.

For example:

(i) π is approximately 22/7, $\sqrt{2}$ is approximately 141/100; and

(ii) 22/7 is approximately 3, while 397/400 is approximately 1.

We now ask how good an approximation to a prime number can be obtained using fractions of a restricted kind.

Case (i). If a_1,a_2,b_1 and b_2 are integers less than a given prime number, p, the minimum of

$$d_2=\left|\frac{a_1.a_2}{b_1.b_2}-p\right|$$ is known to be obtained

when $a_1=a_2=p-1$, $b_1=1$, and $b_2=p-2$. Elementary algebra shows this minimum value to be $1/(p-2)$; which may be used to verify the accuracy of any general program written in response to subsequent cases. Thus (12)(12)/((1)(11)) is the best approximation to 13 by fractions of the given type, and differs from it by 1/11.

Case (ii). If a_1,a_2,a_3,b_1,b_2, and b_3 are integers less than a given prime number, p, we wish to minimise

$$d_3=\left|\frac{a_1.a_2.a_3}{b_1.b_2.b_3}-p\right|$$

For example, if $p=13$ then $a_1=a_2=a_3=10$, $b_1=1, b_2=7, b_3=11$, yields the minimum d_3 value of 1/77.

Note. This result should also be used as a test case for any computer program generated in response to the more general problem posed below. There is a conjecture that the minimum of d_3 approaches $1/p^2$ as p tends to infinity: (becomes larger and larger). This case was investigated at Los Alamos c1960 for all prime numbers less than 100,000 — the evidence is consistent with the conjecture.

Problem

Case (n). $n>2$. Here we ask the question how good an approximation is $(a_1a_2a_3 \ldots a_{n+1})/(b_1b_2b_3 \ldots b_{n+1})$ to the prime p, where the a_i and b_i are all restricted to be less than p.

Section (a). Investigate this for a range of n and small values of the prime p, such as the first ten given in the introduction.

Section (b). Select a particular n, chosen with reference to the above results and extend the range of p, attempting to conjecture the limiting behaviour of d_{n+1} as p tends to infinity.

Readers are invited to submit a program, or suite of programs, to investigate this intriguing approximation problem.

All submissions should include program listings, hardware descriptions, run times and output. They will be judged for accuracy, originality and efficiency (not necessarily in that order). A prize of £10 will be awarded to the 'best' entry received. Entries, to arrive by 1 March 1984, to Mr M R Mudge, Room 560/A, Department of Mathematics, University of Aston in Birmingham, Gosta Green, Birmingham B4 7ET.

Harshad Numbers August

Several enquiries showed an interest in this problem. However, no real progress can be reported. Did holidays interfere with computing, since subsequent problems have been very well supported?

I would ask any interested reader who has neither access to *PCW* August 1983, page 108, nor *Repunits and Repetends* by Samuel Yates, 1982, pages 111-112, or who would simply like further discussion to contact me directly, home telephone (0902-892141). Those who have now got results should please submit them by 1 March, 1984, when a deferred prize award will be made.

Note. Submissions can only be returned if a suitable stamped addressed envelope is provided.

NUMBERS COUNT

Kaprekar Numbers

New readers start here. The topics dealt with in this column attempt to reach the frontiers of knowledge in number theory with the minimal background information. The problems posed therefore have no complete solution known to the author, and readers are encouraged to submit their attempts at solution, however incomplete they may seem.

(1) The legendary number theorist, D R Kaprekar, of the Indian Institute of Science at Bangalore, is perhaps most famous for publicising the number 6174, which is the eventual result *(with certain exceptions)* of the subtraction of a four digit integer whose digits are arranged in ascending order from the same integer whose digits are arranged in descending order.

For example, given the four digit integer 8923 we proceed thus:
9832 − 2389 = 7443,
9963 − 3699 = 6264,
7641 − 1467 = 6174,
7443 − 3447 = 3996,
6642 − 2466 = 4176, the process repeats at this stage.

Question A. What are the certain exceptions referred to above?

Question B. What happens to integers with other than four digits?

(2) Kaprekar Numbers, however, are defined to be those n-digit integers, K, which are equal to the sum of the integer defined by the least significant (right-most) n-digits of their square plus the integer defined by the remaining digits.

Thus 142857 is a Kaprekar Number because here n = 6 and

(i) $(142857)^2 = 20408\underline{122449}$

(ii) 122449 + 20408 = 142857

Question C. What are the Kaprekar Numbers less than 10^n for a given n?

Interest Note. The square of any cyclic permutation of a K-Number is also a cyclic permutation when the digits are added as required.

For example, $(428571)^2$ = 183673102041 and 102041 + 183673 = 285714.

Readers are invited to submit a program, or suite of programs, to answer the above questions. All submissions should include program listings, hardware descriptions, run times and output; they will be judged for accuracy, originality and efficiency (not necessarily in that order).

A prize of £10 will be awarded to the 'best' entry received. Please address all entries, to arrive by 1 April, to Mr M R Mudge, Room 560/A, Department of Mathematics, University of Aston in Birmingham, Gosta Green, Birmingham B4 7ET.

Review—September 83

The Partitions of a Positive Integer generated a very heavy, and varied response; the extremes being typified by an estimate of 15 years to find p(100) using a TI59 programmable calculator to an offer to calculate p(535) programming in Algol 68 — on an unspecified mainframe, one suspects.

A reference work in this field is provided by GE Andrews, *The Theory of Partitions, Encyclopedia of Mathematics and its Applications,* Volume 2, Addison-Wesley 1976. However, the presentation in Chapter XIX of the fifth edition of *An Introduction to the Theory of Numbers, by* GH Hardy and EM Wright, Oxford University Press 1979 is adequate to yield the recurrence relationship for p(n) without which realistic computing is virtually impossible.

$$p(n) - p(n-1) - p(n-2) + p(n-5) + \ldots + (-1)^k\, p(n-\tfrac{1}{2}k(3k-1)) + (-1)^k\, p(n-\tfrac{1}{2}k(3k+1)) \ldots = 0$$

To estimate the magnitude of the computation one may use the asymptotic formula of Hardy and Ramanujan 1917,

$$p(n) \sim \frac{1}{4n\sqrt{3}} \exp\left(\pi\sqrt{\frac{2n}{3}}\right)$$

The computational difficulties are referred to in *Computers in Number Theory* edited by AOL Atkin and BJ Birch, Academic Press 1971.

The prizewinning entry is from RB Shepherd of 2 Orchard Croft, Cottingham, Humberside HU16 4HG using Pro-Pascal on a Sharp MZ80-B computer (64 kbytes). This submission factorised up to p(300) in 44hours 20mins. It must be observed that the presentation of results by RB Shepherd, and indeed by numerous other contributors, was of the highest possible standard. Congratulations all round, and keep the entries flowing.

Note. Submissions will only be returned if a suitable stamped addressed envelope is provided.

END

LEISURE LINES

by J J Clessa

Quickie

A 20lb monkey hangs from a rope which passes over a single pulley and is balanced by a 20lb weight hanging from the other end of the rope. The monkey begins to climb up the rope. Will the 20lb weight go up, go down — or stay still?

Prize puzzle

Three artists — Albert, Brian and Charles — each paint a picture on a square canvas. Brian's picture is seven square feet more than Albert's in area, and seven square feet less than Charles'. All sides have exact measurements. What are the dimensions of the pictures?

November prize puzzle

The puzzle would have presented very little problem for those of you with micros, although a little thought would have saved quite a bit of computer time — especially if you were to generate *all* 10-digit numbers rather than the smallest.

Since the required answer had to be divisible by the integers 1 to 12, it must be a multiple of 12 x 11 x 7 x 5 x 3 x 2 — that is 27720. Hence you could set up a simple loop with increments of 27720. Luckily, the solution is 1 234 759 680, which comes up relatively early in the loop.

You can cut down considerable time by tailoring the loop to skip whenever the second digit is equal to the first.

The winning entry came from D Spencer of Ruislip, Middlesex whose prize is on its way.

To those overseas entrants who were worried that their entries might arrive too late: I can't recall a case in which a late entry was from overseas.

END

Feb 1984

NUMBERS COUNT

Number theories

New readers start here. The topics dealt with in this column attempt to reach the frontiers of knowledge in number theory with the minimal background information. The problems posed therefore have no complete solution known to the author, and readers are encouraged to submit their attempts at solution, however incomplete they may seem.

This month we look back and examine some number theoretic results which were making the news around the turn of the century. We ask how these might be established using a digital computer, and in what ways they may be extended.

The results, which require no understanding of mathematics beyond elementary arithmetic, are given in chronological order and readers are invited to respond to some, or all of them!

(a) In 1876 AB Evans found four integers whose sum is a sixth power, and such that the sum of any three is a fifth power.

(b) In 1895 several writers found two integers whose sum, difference and difference of their squares are all twelfth powers.

(c) In 1898 GBM Zerr found six positive integers x_1, x_2, x_3, x_4, x_5 and x_6, such that each, diminished by $(5/2)(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)^5$ becomes a fifth power, and three years later three numbers in arithmetic progression (A, A+d, A+2d, where A is the smallest and d the common difference of arithmetic progression) whose sum is a sixth power.

(d) In 1904 P F Teilhet verified that every integer, A, up to 600, with one exception is a sum of two squares and two positive or zero cubes.

(e) In 1917 R Goormaghtigh stated: 'For A less than 1000000 $A=1+x+x^2+x^3+\ldots+x^m=1+y+y^2+y^3\ldots+y^n$ holds only in two cases, one of which is $31=1+5+5^2=1+2+2^2+2+2^4$.'

Readers are invited to submit a program, or suite of programs, to recreate the news items listed above, and to extend them in any way. Thus in (d) the exception should be displayed and, hopefully, the bound on A significantly extended, while in (e) the second case should be found explicitly and, again, the bound extended on A.

A prize of £10 will be awarded to the 'best' entry received by 1 May, 1984. Please address all entries to Mr M R Mudge, Room 560/A, Department of Mathematics, University of Aston in Birmingham, Gosta Green, Birmingham B4 7ET.

Note. Criteria of judgment include limitations imposed by hardware and programming language chosen, so details of these should be supplied.

Review — October 83

The concept of a *Perfect Number* appears to be well known to many *PCW* readers, hence the peak in response to this article.

The problem of finding the factors of an integer is recognised by many, but few address themselves to it. The use of the Chinese Remainder Theorem may provide a 'best possible' algorithm, but personally I doubt this. Discussions with Dr R Churhouse and Dr AOL Atkin many years ago at the Atlas Computer Laboratories, Chilton, in the presence of my colleague Dr D Ridout, revealed that there was still an unsolved problem in this area.

The Poulet sequence has been displayed by many correspondents — some suggested problems include:

(1) Are all abundant numbers divisible by 15 if they are odd?

(2) Are there any three-ply Perfect Numbers different from 120 and 672?

Therefore, does there exist an n for which $\sigma(n)=kn$, k greater than 2 defines a k-ply Perfect Number.

The winner is Mr J Jones, 33 Vincent Avenue, Nantgylo, Gwent NP3 4PF. He has taken this problem to the limit of his available hardware, changing the programming language on the way.

Once again, it should be observed that many submissions were of the highest possible standards of neatness; however, I claim to recognise valuable work among the other submissions, and ask that you are not put off by the lack of a word processor.

A number ^{m} is said to be:

2-hyper-perfect if $m=2s(m)-1$

3-hyper-perfect if $m+3s(m)-2$; and in general

n-hyper-perfect if $m=n\,s(m)-(n-1)$.

D Minoli has constructed (1980) a list of all n-hyper-perfect numbers (n greater than 1) up to 1500000 using the PDD 11/70 computer.

Can anyone improve upon this situation?

Note. Submissions will only be returned if a suitable stamped addressed envelope is provided.

END

LEISURE LINES

by J J Clessa

Quickie

Of 50 people interviewed 27 liked cricket, 32 liked soccer and five didn't like either. How many people liked both cricket and soccer?

Prize puzzle

The Smiths and Jones families each have three children who work for the local authority. By coincidence, the salaries of the three Smith children are in the same proportions as those of the three Jones children.

Moreover, Albert Smith and Paul Jones have the same salary — as do Mary Smith and Sally Jones. However, Peter Jones earns £387 per month more than Michael Smith. What is the salary of each?

Answers please — postcards or backs of sealed envelopes only — to reach *PCW* not later than last post on 30 March, 1984. Send your entries to *PCW*, March prize puzzle, Leisure Lines, 62 Oxford Street, London W1.

December prize puzzle

Not too difficult to solve analytically, although it's a bit harder to solve by trial and error using a micro. Naturally, it isn't possible to say which of the two scores is for the penalty and which is for the field goal, but the scores must be 4 and 17.

This prohibits scores of:
1 2 3 5 6 7 9 10 11 13 14 15 18 19 22 23 26 27 30 31 35 39 43 and 47 — twenty-four in all.

The winning entry, drawn at random from over one hundred, came from C R Hensler of Letchworth, Herts. Congratulations, Mr Hensler, your prize is on its way.

Don't forget — please send entries on postcards (backs of sealed envelopes will do). Letters are immediately disqualified.

END

NUMBERS COUNT

Repunits

This month Mike Mudge takes a long look at Repunits and Repdigits

The first part of this month's problem, although very simple to formulate, should encourage the development of certain general integer length arithmetic routines. See, for example, DE Knuth's, *The Art of Computer Programming*, Vol 2, Semi-numerical Algorithms, Addison Wesley, 1969; such algorithms once optimised will prove invaluable in any future empirical number theory.

The second part, somewhat tenuously related to the first, is in response to numerous requests for further problems relating to Prime Numbers; and is an opportunity to mention the possible sinister significance of such numbers in 1984, hinted at by A Berry, *The Daily Telegraph*, 9 January, 1984 together with the paper *The Fascinating Hunt for Prime Numbers* by C Pomerance in *The Scientific American*, December (1982).

1) Defining a Repunit by $R_n = (10^n - 1)/9$, an integer consisting of a string of n 1's. The problem is to factorise R_n completely for a given n. Thus $R_2 = 11$, $R_3 = 3.37$, $R_4 = 101.11$, $R_5 = 271.41$.
2) Noting that Repdigits (defined in the obvious way) other than Repunits are always trivially composite (not prime), it is known that in common with Repunits they can occur as long strings in primes. Thus R_{317}, 2222222222 2222222222 39, 33333333333333333333 01, 1733333333 3333333333 33, 4444444444 4444444444 51 are all primes!

Furthermore, $10^{564} + 10^{282} - 1$ consisting of 1 followed by 282 0's and 282 9's is also prime.

Find primes containing lengthy Repdigits 5,6,7 and 8.

It is likely that this section will involve considerable library work and hopefully not too much computing, as a variation on the usual balance between these two activities in Numbers Count.

A prize of £10 will be awarded to the 'best' entry received by 1 June, 1984. Please address all entries to Mr MR Mudge, Room 560/A, Department of Mathematics, University of Aston in Birmingham, Gosta Green, Birmingham B4 7ET.

Note. Criteria of judgement include limitations imposed by hardware and the programming language chosen, so details of these should be supplied.

The Persistence of an Integer Review — November 1983

The Persistence of an Integer provided a popular challenge: with typical results to (c) examining powers up to 2^{9764} radix 3 in about 32 hours of Basic on a BBC Micro.

Parts (d) and (e) are still very much closed books and results relating to them would most certainly be of interest to myself and to this month's prizewinner, Mr Alan Prior of 41 Walnut Tree Road, Shepperton, Middlesex.

Alan used Basic on his Sharp MZ-80A with 48k and a 2MHz processor, having first rejected Pascal and Forth: the former due to the limitations of his version; and the latter due to lack of time to become familiar with the language.

In six hours two minutes (and eight seconds) he established 2777777 88888899 as the smallest number with persistence 11 using a program which handles 78 digit integer input and 255 digit integers internally.

Attempts to find the smallest integer with persistence 12 have so far been unsuccessful, although tables of persistence of n for n = (1) 24999, if extended, may shed light on this problem, should some underlying pattern be revealed.

The origins of this problem, to the best of my knowledge, are to be found in NJA Sloane's *The Persistence of a Number, Journal of Recreational Mathematics*, Vol 6 1973 (pp 97-98).

Note. Submissions will only be returned if a suitable stamped addressed envelope is provided.

LEISURE LINES

by J J Clessa

Quickie

If 250 players enter a darts knockout tournament, how many matches will have been played by the time the tournament is finally won?

Prize puzzle

Can you complete the 3×3 grid shown

	C	P	S
S			
P			
C			

here so that the row and column marked 'P' contain a prime number, those marked 'C' contain a perfect cube, and those marked 'S' contain a perfect square.

January prize puzzle

A very mediocre response to the January puzzle, about 50 entries only. Perhaps it was more difficult than usual, or maybe my readers are spending their time on the puzzles to win the Apricot.

However, many of those who did submit entries found quite a lot more to the puzzle than I realised. Clearly, there are many solutions, the smallest of which (with 7 digits) is 1000146, whose divisors total 2286144 (1512^2). The largest is 9998508 which is 5040^2.

The winner was chosen by a draw, and the lucky entrant was from Milan, Italy — Mr Giorgio Vincenti. He only submitted one solution, 1380527, whose divisors sum to 1176^2 but it was enough to win the prize. Congratulations, Giorgio, your prize is on its way.

Incidentally, solutions should always be submitted on postcards or the backs of sealed envelopes. Normally solutions on letters are ineligible for prizes, but since we forgot to state this in the January puzzle we let it go by this once. But don't forget — letters if you want to correspond, postcards for the puzzle entries.

STOP PRESS

Amstrad Benchtest – (p170)

The original marketing plan to keep the basic machine as bare and plain as possible, with absolutely no accessories, has been rethought and a disk will be available around July.

The result will be that the Amstrad will be the cheapest CP/M micro on the market.

There will be a £400 system with a green monochrome monitor and a single Hitachi drive, including CP/M 2.2 plus the programming/teaching language, Logo, also from Digital Research.

And for £500, the same system, but with a colour display, will be available as a 'top of the range' model.

It was not possible to obtain a sample disk for the Benchtest; this will be covered in a future issue of *PCW*. However, the Hitachi drives are known to be reliable, and the only drawback is that there are many programs on CP/M which don't come readily available on 3in diskettes.

However, once there are a few thousand Amstrad systems with CP/M, I predict that there will suddenly be an interesting supply of software for the new format.

Guy Kewney

NUMBERS COUNT

Diophantine Equations

The topics dealt with in this column attempt to reach the frontiers of knowledge in number theory with the minimal background information. The problems posed therefore have no complete solution known to the author, and readers are encouraged to submit their attempts at solution, however incomplete they may seem.

Those readers who have been with us since the first Numbers Count back in February 1983 — 'Waring's Conjecture and a certain Diophantine Equation' — will recall that a Diophantine Equation is one which is solved in terms of integers only.

The first writer to study such equations in detail was Diophantus of Alexandria *c* 250AD. For example, the equation $x^2\ y^2 = z^2$ yields the integer sided right-angled (or Pythagorian) triangles beginning with (3,4,5) and (5,12,13).

Problem

Here are three distinct problems in this field, indicating fundamental differences in the state of the art relating to each. Readers are invited to contribute.

(1) Consider $z(1 + xy) = x^2 + 2y^2$; this has only one known solution in integers, namely $x = 30905$, $y = 663738$, $z = 43$ due to ES Barnes, *J London Math Soc* Vol 28, 1953 pp242-244. Further, LJ Mordell in *Diophantine Equations*, Academic Press 1969 writes: 'The only procedure seems to be to try if there is a solution for various values of z.' How does one best do this trying, and do we need all values of z?

(2) Consider $6y^2 = (x+1)(x^2 - x + 6)$ (those readers familiar with the Binomial Theorem will recognise this as $y^2 = 1 + x + x(x-1)/2! + x(x-1)(x-2)/2!$). This is known to have integer solutions for x=2,7,15 and one other non-trivial value of x (x=0, and x=−1 are regarded as trivial). Find the fourth non-trivial x-value: it has only two digits — are there others?

(3) The Arabs *c* 972AD are believed to have been the first to study the pair of simultaneous Diophantine Equations

$$y^2 = x^2 + 5u^2.$$
$$z^2 = x^2 - 5u^2$$

The solution x=41, y=49, z=31 and u=12 was published by Leonardo of Pisa 1220AD. A further solution x = 3444161, y = 4728001, z = 113279 and u = 1494696 is known, as is a yet larger solution involving 15-digit integers.

Theoretically, this problem is completely solved because algebraically every solution may be derived from Leonardo's by rational operations. See Uspensky and Heaslet, *Elementary Number Theory*, McGraw Hill 1939 pp419-427.

How efficiently can the above solutions be found using a computer? Readers are invited to submit a program, or suite of programs, to investigate the above questions. All submissions should include program listings, hardware descriptions, run times and output; they will be judged for accuracy, originality and efficiency. A prize of £10 will be awarded to the 'best' entry received by 1 July 1984. Please address all correspondence to Mr MR Mudge, 'Square Acre', Stourbridge Road, Penn, Nr Wolverhampton, Staffs WV4 5NF.

Absolute differences of Prime Numbers — December 1983

This problem proved to be exceptionally popular, attracting multiple responses from Belgium and West Germany. The languages chosen included VSAPL under CMS in a 2Mbyte virtual machine of a 4Mbyte IBM4331/2; Pascal on an Altos ACS 68000 with the Unix System III in multi-user mode; C-language on an IBM Personal Computer; Basic on an Acorn Atom with 29k of RAM but with a 7-track ½in tape drive interfaced to give mass storage with a transfer time of around 4k per second.

The prizewinner however, after a very careful evaluation, is Michael Robinson of 2 Lower Merrion Street, Dublin 2 who addressed himself precisely to the problem as posed. Using Cobol written for a 16-bit micro, with assembly routines for the repetitive parts, the program was ultimately run on a Burroughs B22 up to $a_{110} = 103961$ and then in mortuary time on a B21. A very careful operations estimate was included and the entire study well documented. $a_{64} = 5940$ was reached in 4mins 42secs from approximately 6000 primes, the study being terminated at $a_{146} = 733576$ in 27hrs from 786575 primes, the last of which was 11975597. Empirical evidence for the Gilbreath conjecture is considerably strengthened by this computation, revealing, for example, that around $a_{126} = 271621$ large differences are seen 'spreading like ripples in a sea of 0s and 2s.'

Perhaps those who submitted studies of this problem could communicate one with another either via Michael Robinson or myself, with a view to a final assault on the a_n and its associated number patterns? END

Note. Submissions can only be returned if a suitable stamped addressed envelope is provided. Telephone comments, both favourable or otherwise, are welcome on (0902) 892141.

May 1984

printer cables, but Tradecom says it can produce only 25 per week.'

Brainwave will still deliver to customers who have already ordered but is directing all new business to Tradecom UK. Details on (01) 941 3519.
Jerry Sanders

'C' is for Apple

Apple Computer has finally launched its much rumoured transportable version of the Apple IIe in the form of a 7½lb machine that costs £925 plus VAT. It should run almost all current IIe software.

Called the IIc, the one-piece machine comes as standard with a built-in, half-height 5¼in disk drive, 128k RAM, and an RF modulator to allow the machine to connect up to a television set. It is based on the CMOS version of the 6502 chip, known as the 65C02. A simple, typewriter style, keyboard is

Similarly, if you want to use the IIc with its optional 9in Apple monitor (£140 extra), another button just above the featured but this can be converted from the standard qwerty keyboard layout to the alternative Dvorak keyboard style at the touch of a button.

keyboard allows the user to switch from 40-column to 80-column display mode. Original Apple II users still upset about past mistakes with the one-touch reset button will be glad to hear that although the IIc's reset button is set to the top left of the keyboard, it must be depressed with two other special keys before it will operate.

A range of I/O connectors identified by icons are built into the back of the IIc. One connector has been provided to allow a mouse — or a joystick — to be attached (many new packages are being developed for the IIc that are mouse compatible and old ones are being enhanced for the same reason).

The IIc mouse will cost £70.

Other connectors include two RS232 serial ports for connecting a modem and/or a printer, an extended video port for attaching the RF modulator or a special colour monitor, a standard video port, and a second disk drive port. The IIc is powered by an external transformer, but it is not too choosy about where the DC power comes from — this means that you could operate it off a car cigarette lighter socket if you have a battery-powered television. By the end of the year Apple expects to announce a liquid crystal display that clips onto the IIc and which can handle 80 columns by 24 lines in what is being called super or ultra high resolution. This is a far more expensive proposition than a TV since it may cost as much as £600.

Except for what you can add via the I/O connectors, the IIc is not expandable in the conventional sense, and Apple hopes that this all in one approach will appeal to those wanting to buy a computer for use in the home.

Sandwiching it on either side are various Apple IIe system bundles: it will be possible to buy a special IIe starter system for just £795 (CPU, one disk with controller, no display); a professional system for £1095 (two drives and a monitor); and a business system for £2290 (this is a professional system plus a 5Mbyte Profile hard disk.
Robin Webster in the US

Apricot delight

Is your Apricot screen giving you eye-strain? If so how about a 12in monitor?

Emco Electronics expects the idea to appeal and is offering to supply a customised Indesit monitor (green on black) for around £220.

From June new Apricot buyers can specify the 12in screen to dealers on ordering, but if your dealer pleads ignorance, get in touch with Emco's marketing manager, David Bernheim, on (01) 737 3333.
Jerry Sanders

PCW Subscriptions down!

Relentless diatribes from* PCW *subscribers and the editorial team have finally worn down your publisher. He has agreed in his magnanimity and mortification to cut the subscription cost of the greatest micro magazine from £25 to £15 per year.

***If you've already forked out a whole £25 for* PCW *subscriptions, don't have apoplexy — you are now entitled to a whole extra year of* PCWs.**

NUMBERS COUNT

Quotients

This month Mike Mudge examines the quotients of Fermat and Wilson.

Notation

We write $A \equiv B \pmod C$, read as A is congruent to B modulo C, if A and B leave the same remainder when divided by C. For example, $16 \equiv 21 \pmod 5$, $64 \equiv 0 \pmod{16}$.

The Fermat Quotient

A famous theorem in classical number theory, Fermat's Little Theorem states that if p is prime and does not divide a, then $a^{p-1} \equiv 1 \pmod p$. The number $F_p = \frac{a^{p-1}-1}{p}$ is Fermat's Quotient.

Computations by Fröberg for a=2 and p less than 5000 have revealed only two solutions, p=1093 and p=3511, to the equation $F_p \equiv 0 \pmod p$. It is still an open question whether any more solutions exist for a=2 with p greater than 5000, and the nature of any solutions for $2 < a \leqslant 31$; this range being relevant to the proof of Fermat's Last Theorem.

The Wilson Quotient

A well-known theorem by Wilson states that if p is prime, then $(p-1)! \equiv -1 \pmod p$. The number $W_p = \frac{(p-1)!+1}{p}$ is known as Wilson's Quotient.

Computations by Fröberg for $3 \leqslant p \leqslant 50000$ have revealed only three solutions, p=5, p=13 and p=563, to the equation $W_p \equiv 0 \pmod p$. It is theoretically probable that there are more solutions to this equation.

Large tables of $W_p \pmod p$ would be useful in empirical number theory. At the present time, only two small tables are known: p less 300, and p less than or equal to 211, due to NGWH Berger (1920) and ET Lehmur (1937).

Readers are invited to reproduce the results given above, and to extend them in any natural way. Submissions should include program listings, hardware description, run time and output, and will be judged for accuracy, originality and efficiency (not necessarily in that order). A prize of £10 will be awarded to the 'best' entry received by 1 August 1984. Please address to Mr MR Mudge, 'Square Acre', Stourbridge Road, Penn, Nr Wolverhampton, Staffs WV4 5HF.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided.

Fractional Approximation to Primes Review — January 1984

This problem was attacked by a number of readers. A careful analysis of the results shows that a ZX81 with 16k RAM has produced the 'best' result. Mr L Faltin, Wilhelminenstrasse 147/1, A-1160 Wien has compared orders of approximation (. . .5). The use of log/log graph paper may shrink the computing time, apparently!

Decomposition of p=31 to the 5th order may need about four days.

Is this statement true or false? Answers to Mr Faltin or Mike Mudge.

Brun's Constant

The sum of reciprocals over the twin primes converges to a finite limit, known as Brun's Constant. Ed Rosenstiel decided to attempt the calculation on a micro, and made some interesting discoveries in the process.

$25,000 Prize

Worldwide Computer Services is offering a $25,000 prize until 31 March 1987 to prove or disprove that there are infinitely many twin primes (the twin prime conjecture).

Ed Rosenstiel's article illustrates how far down the road you can get with a micro today; previously the calculations shown have been done with minis and mainframes.

One learns at school that the so-called harmonic series '1 + 1/2 + 1/3 + 1/4 + 1/5 + . . . and so on' diverges to infinity, *but so does* 1/2 + 1/3 + 1/5 + 1/7 + 1/11 + 1/13 + 1/17 . . . , that is, summing similarly but only over the primes.

Schur demonstrated this in a lecture in 1932 in Germany as follows:
Assume the contrary: that is, that the sum of the prime reciprocals converges to some limit, say, K.

Then, by a formula due to Euler, we have 1 + 1/2 + 1/3 + 1/4 + 1/5 + . . . + 1/n $< (1 + 1/p_1 + 1/p_1^2 + 1/p_1^3 + ..) * (1 + 1/p_2 + 1/p_2^2 + 1/p_2^3 + ..) * ... * (1 + 1/p_m + 1/p_m^2 + 1/p_m^3 + ..)$
where the p_i on the right-hand side are just the m prime factors of all numbers from 1 to n.

A little bit of simple calculus then shows that *for all n:*

$$1 + 1/2 + 1/3 + 1/4 + 1/5 + .. + 1/n < \prod_{i=1}^{m} 1/(1 - 1/p_i) < \prod_{i=1}^{m} e^{2/p_i} < \exp[2*(1/2 + 1/3 + 1/5 + 1/7 + 1/11 + 1/13 + 1/17 + \dots \text{to infinity})] = e^{2K}$$

by the assumption, so the RHS is finite. Thus the sum of the reciprocals of all positive integers is also finite, which is false. Hence, so was the assumption. Therefore the sum of the reciprocals of *all* the primes also diverges to infinity!

Then Schur tantalised his audience by mentioning some of the problems connected with the so-called twin primes (3,5), (5,7), (11,13), (17,19), namely:
(i) it was an unsolved problem (*and still is!*), as to whether the list of twin primes ever ends; and
(ii) in 1919 Viggo Brun (who died only recently at the age of 92) stunned the mathematical world with a proof that the sum of reciprocals not over *all* the primes, but only over the twin primes (even if their number could be shown to be infinite) converges to a finite limit which is now known as **Brun's Constant** = say, S.

This much I remembered when, as part of a computer course at Birkbeck College in Pascal, I embarked on a project to calculate Brun's limit.

Writing a program in Basic to list twin primes and to evaluate the sums of their reciprocals is not difficult. The problem is that to find all the twins there is no other way but to compute almost *all* the primes, and this is a slow business on any computer. On a Commodore PET (since the machines operating Pascal were too busy most of the time), I went up to the last pair under 3020001, (later extended to 5000001), then made a

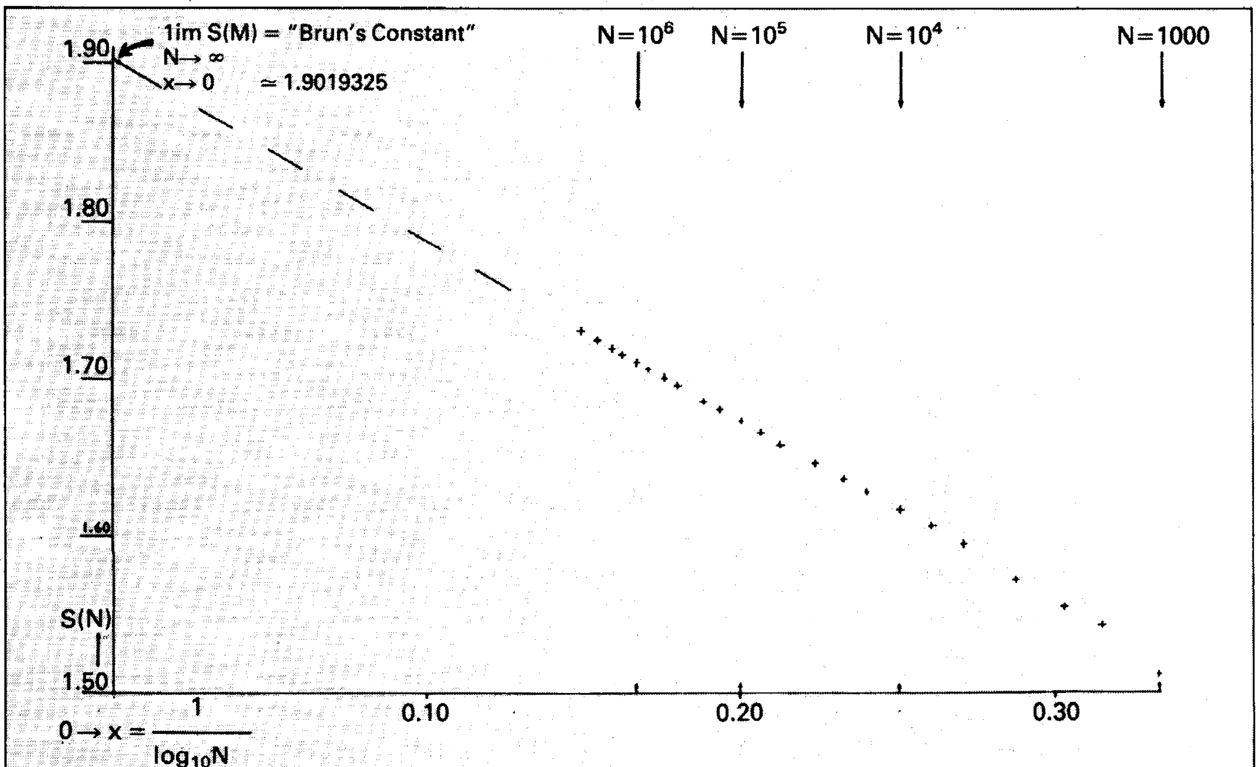


Fig 1 Graph of $x = 1/\log_{10}N$, $y = S(N)$ for N = 1000 to 5000001 gives an estimate for Brun's Constant $S \simeq 1.9019325$.

Jul 1984

graph of necessity in logarithmic scale: that is, in powers of 10. It looked irregular at the lower end, but the gentle curve for the higher values looked promising and I also remembered that, according to Brun's Theorem, this curve would approach some horizontal line for very high values.

It seemed a good idea to eliminate the logarithmic scale, so I plotted 1/log N instead of N on the X-axis, and also left out the lower values under 10000 (Fig 1) and a *straight line* appeared.

It is remarkable in the wilderness of prime numbers, to come across an *apparently* straight line. Ignoring a professional mathematician's remark: '. . . if you take any kind of data and keep taking logs often enough, you will end up with a straight line. . .', my instinct told me this might be something original.

Using a TI-59 program which works out the least squares fit of a polynomial, I soon confirmed that I had found a much more accurate straight line than had it merely been deduced from a graph (Table 1a). And some extrapolations to values higher than those used for the least squares approximation were later found to agree with their computer counterparts to four significant digits!

Looking seriously at what was behind these findings, I decided to retrace the steps which had led me to such an extraordinary result: the 'gentle curve' prompted me to look for some closed mathematical expression to graph it and I had noticed that:

a) it was convex; and

b) it was asymptotic to a line parallel to the x-axis by Brun's Theorem, so I had thought of curves which might fit. By chance I had hit on the right answer straight away, namely on $y = S - 1/x$, the 'upside down hyperbola', although I had meant to consider also $y = S - 1/\exp(x)$ if $y = S - 1/x$ would not work.

The next step was to make a thorough literature search. Brun's Constant had indeed been calculated by several workers (3,4), and the most recent *probable* value given (4) was: $1.9021604 \pm 5 * 10^{-7}$

However, all the calculations had assumed that the famous conjecture made in 1923 by Hardy and Littlewood (6) is true. This says that the number of twin prime pairs up to some number X is closely approximated by:

$L_2(X) = 2c_2 *_2 \int^{X} dt/(1n\,t)^2 \sim 2c_2 X/(1n\,X)^2$

that is, neglecting terms of order $X/(1nX)^3$, where $c_2 = 0.66016181$ . . is the 'twin prime' constant as given by Brent (4).

Furthermore, Brent estimates, making the assumption that twin primes are randomly distributed with density $2c_2/(1nx)^2$, (which implies that Brun's series is an infinite series):

that $1im\ S(n) — S(X) \sim 4c_2 *{}_x\int^{\infty} dt/$ $n \to \infty$ $t*(\ln t)^2 = 4c_2/1nX$

which is the 'Straight Line Conjecture' that I had come up with on the PET (Table 1b), with $c_2 \simeq 0.25*k*1n10 = 0.6596417$ . . .

Does this show that, 60 years after two brilliant mathematicians had deduced a (so far unproven, but, in practice, very accurate) formula for the number of twin primes, by taking the opposite route, from the Straight Line Conjecture to the Hardy-Littlewood approximation, a mere tyro *could* have discovered this celebrated formula on a micro?

Computations

All computations were done on a Commodore PET with a simple program. These were cross-checked on a faster 'sieve' program which leaves out division by multiples of the first primes 2,3,5,7, and 11, and other checks were made against printouts of primes from a TI-59 calculator.

Most results were just copied from the VDU, but a complete printout of all twin primes less than 100000 allowed a manual count of 1224 in agreement with figures previously published by Brent (4). It was interesting to compare the calculation speed of the sieve with that of the simple program: it took the latter 25.3 days to reach the twins up to N = 1700000, while the sieve program needed only 12.2 days, a saving of ≃52%! (The sieve program took 54 days for a complete run up to N + 5000001.)

From the least squares fit (Table 1a) it will be seen that the value derived for S, on the assumption that the Straight Line Conjecture is true:

that Brun's Constant

$S = 1im\ S(N) = S(N) + k/\log N +$ error (N), $N \to \infty$

is $S = 1.90074$ . .

which agrees with Brent (4) for three significant digits,

while from $k = 1.1396 . . \simeq 4c_2 * 1n10$

we have $c_2 \simeq 0.6560$ . . .

However, there is something rather unsatisfactory in the above approach, where values below some arbitrarily chosen N are ignored for the extrapolation to S, and it is then observed that all higher values appear to lie on a straight line — not exactly, but to a high degree of 'accuracy'. (This mimics the quite different situation in physical experiments, where data is inevitably tainted due to to observational errors.) I was thus led to consider the question whether 'better' estimates for Brun's Constant might be obtainable by using a statistical approach to curve fitting.

With the help of the Applied Statistics Module for the TI-59 (7), I re-evaluated the results obtained, and also computed the correlation coefficient 'r'. Next I tried to *improve* 'r' by excluding in turn one value, arguing that because of the locally irregular distribution of primes one particular value might perhaps unduly influence the final result. As was not altogether surprising, the coefficient was improved by omitting either of the two *lowest* values for N, so I felt justified to omit both and to start calculating from N = 100001 upward, using higher values for S(N), which had come to hand. From Table 1b N = 734001 was omitted when calculating the final figures. These were: S = 1.901932526, k = 1.14591496., therefore $c_2 \simeq 0.6596417.$, where c_2 differs by 0.079%, S by 0.012% from the published results already mentioned. (The correlation coefficient was: r = 0.9999908.)

Conclusion

What I called the Straight Line Conjecture is not new, but during simple micro computations it suggested itself in a most obvious way; yet there was no hint about how to estimate *independently* the errors with these methods. If one uses the most recently published estimates for S and c_2 to calculate error terms for each N of Table 1b; that is, error (N) = $S - S(N) - 4c_2/_{1nN}$, then by a simple calculator exercise we have: $|\text{error}(N)| < 2/_{N}{}^{0.66}$, so $k/1nN$ dominates the approximation.

An essential difference between Brun's and other converging series is seen when comparing it with Gregory's well-known series (which was also discovered independently by Leibniz): $\pi = 4[1 - 1/3 + 1/5 - 1/7 + - . . - 1/(2n-1)] + 1/n +$ error(N), where the error consists of terms of the form constant$/_n(2k+1)$ with $k>0$.

Now the square-bracket expression converges to $\pi/4$ with any desired number of decimals, (although much too slowly without the correction 1/n to be of any practical use), provided that a sufficient number of terms is computed (8). To show that the same is true for Brun's series still requires proofs of conjectures of one kind or another, even if better estimates were obtained for Brun's Constant by the use of more powerful computers. It will be remembered that to determine S to only three significant figures by computing its partial sums, requires a program to 'look' at *all* prime numbers up to 10^{1000}.

Until new theories are discovered, one can still only make 'plausible' estimates, — however well these might seem to fit with computation carried out so far.

Thus the mysteries of Brun's series still beckon: only one of the many unsolved problems of The Theory of Numbers.

It is not known whether Brun's converging series S = 1/3 + 1/5 + 1/5 + 1/7 + 1/11 + 1/13 + 1/17 + 1/19 + 1/29 + 1/31 + . . . + . . has an infinite number of terms, but if so then it probably converges *very slowly* indeed with the largest error term ≃ 2.64/1nN. This has been compared with Gregory's infinite series for π which has as largest error term 1/N, thus converging too slowly for practical computation, but still much faster than Brun's series. A more well-behaved series (although a rather trivial example) is the geometric series $G = 2 = G(N) + 1/_{2^n}$ with

$G(N) = (1 + 1/2 + 1/4 + 1/8 + . . + 1/_{2^n})$

where the error term is exactly $1/_{2^n}$ and convergence is correspondingly fast.

1984

Table 1a
Plotting S(N) against $\log_{10}N$

N	$\log_{10}N$	S(N)
51	1.708	1.2700
71	1.851	1.3032
101	2.004	1.3310
151	2.179	1.3969
201	2.303	1.4286
301	2.479	1.4602
501	2.700	1.4861
701	2.846	1.5061
1001	3.000	1.5180
1501	3.176	1.5426
2001	3.300	1.5549
3001	3.477	1.5722
5001	3.699	1.5947
7001	3.845	1.6067
10001	4.000	1.6169
15001	4.176	1.6279
20001	4.301	1.6359
30001	4.477	1.6462
50001	4.699	1.6585*
70001	4.845	1.6652*
100001	5.000	1.6728*
150001	5.176	1.6806*
200001	5.301	1.6858*
350001	5.544	1.6953*
500001	5.699	1.7007*
734001	5.866	1.7064
1020001	6.009	1.7111
1142001	6.058	1.7127
1420001	6.152	1.7156
1500001	6.176	1.7164
1700001	6.230	1.7180
1800001	6.255	1.7188
2000001	6.301	1 7201
3020001	6.480	1.7251

......

RESULTS:
From the starred values by the TI-59 pakette(2) program:

$S \simeq 1.90074..$
$k = 1.139594148$
$c_2 \simeq 0.65600..$

Table 1b
Plotting S(N) against $1/\log_{10}N$
where $S(N) = \sum [1/p + 1/(p+2)]$
$p<N$, (p and p+2 prime)

N	$1/\log_{10}N$	S(N)	least squares fit to S(N)
100001	0.1999998263	1.67279958	1.672750.
150001	0.1931958674	1.68055034	1.680546.
200001	0.1886425074	1.68584216	1.685764.
350001	0.1803729262	1.69527377	1.695240.
500001	0.1754702774	1.70071693	1.7008585
734001*	0.1704827337	1.70642789	1.706574.
1020001	0.1664281031	1.71108006	1.711220.
1142001	0.1650800688	1.71268937	1.712765.
1420001	0.1625411382	1.71564571	1.715674.
1500001	0.1619146983	1.71635648	1.716342.
1700001	0.1605020716	1.71802810	1.718011.
1800001	0.1598651315	1.71877363	1.718741.
2000001	0.1587042065	1.72013171	1.720071.
3020001	0.1543208189	1.72513665	1.725094.
5000001	0.1492766778	1.73097675	1.730874.
....			
$2*10^{10}$	0.0970776709	----------	1.7906898.
$1*10^{99}$	0.0101010101	----------	1.8903576.
....			

RESULTS:

$S \simeq 1.9019325..$ [cf. Brent (4) who gives a probable value for S as: $1.9021604 \pm 5*10^{-7}$]

$k = 1.14591496$, hence
$c_2 \simeq 0.6596417...$ and $r = 0.9999908...$

where r is the correlation coefficient computed by the TI-59 Bivariate Data Transform Program ST-12 (6).
(The starred value 734001 was not used for calculating these results,cf.p.7)

IN BOTH TABLES: $S = \lim_{N\to\infty} S(N)$

$S - S(N) \sim k/\log_{10}N$

$k \simeq 4c_2/\ln 10$ and $c_2 = 0.660161181$

[$\simeq$ is used for 'approximately equal to',
$\sim$ means 'asymptotically equal to' in the strict mathematical sense (cf.LeVeque (5)) and
c_2 is the 'twin prime constant' as given by Brent (4).]

References

1 **C L Baker and F J Gruenberger,** *Primes in the thousandth million,* P-1245, The Rand Corporation, Santa Monica, 1958

2 **J Walston,** *Least-mean-square fit of a polynomial,* Texas Instruments Inc, 1978

3 **D Shanks and J W Wrench, Jr** *Brun's Constant,* Math Comp, v28, 1974 pp293-299

4 **R P Brent,** *Irregularities in the distribution of primes and twin primes,* Math Comp, v29, 1975, pp43-56

5 **W J Leveque,** *Fundamentals of Number Theory,* p134, Addison-Wesley Publishing Company, London, 1977.

6 **G H Hardy and J E Littlewood,** *Some problems of partitio numerorum,* Acta Math, v44, 1923, pp1-70

7 **Applied Statistics,** *Solid State Software Module,* Texas Instruments Inc, 1977

8 **C W Kilmister,** *Summing infinite series,* The Gresham Lectures 1983

With thanks to the staff of Birkbeck College, Department of Mathematics.

Ed Rosenstiel came to England from Germany in the 1930s. He was a practising dentist until his retirement in 1978, when he became a maths undergraduate at Birkbeck College, London.

END

Tree-like structures

This month Mike Mudge guides you through the ramifications of tree-like structures due to Collatz.

As a student, the famous numerical analyst L Collatz asked if the sequence defined by:

$a_{n+1} = a_n / 2$ when a_n is even together with $a_{n+1} = 3a_n + 1$ when a_n is odd, is tree-like apart from the root 4,2,1,.

By this he meant, starting from an arbitrary positive integer a_1 and repeatedly applying the appropriate formula chosen from the above pair, is there always a value of n, which we may denote by $c(a_1)$ such that $a_n = 1$?

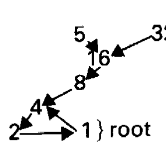
For example: if $a_1 = 10$, then $a_2 \times 5$, $a_3 = 16$, $a_4 = 8$, $a_5 = 4$, $a_6 = 2$, $a_7 = 1$. Thus we write $c(10) = 7$. It is readily verified that $c(152) = 24$ also that $c(18) = 21$.

D H & E Lehmer together with J L Selfridge have answered this in the affirmative for 0 less than a_1 less than 10^9 while as recently as 1978 Corroda Böhm and Giovanna Sontacchi extended the range of a_1 up to 7×10^{11}.

Now if $3a_n + 1$ is replaced by $3a_n - 1$ it seems likely that the possible trees have roots $(\dot{1},\dot{2},.)$ or $(\dot{5},14,7,20,\dot{1}0,.)$ or $(\dot{1}7,50,25,74,37,110,55,164,82,41,122,61,182,91,272,136,68,34,.)$

The generalisation of these two problems is due to D C Kay (Pi Mu *Epsilon Journal*, Vol 5, 1972, page 338) who defines a sequence thus $a_{n+1} = a_n/p$ if p is a factor of a_n else $a_{n+1} = a_n \times q + r$ and asks for what values of p,q and r the problem can be completely analysed?

Readers are invited to reproduce the results given above, and to extend them in any natural way. Particular interest is focused upon computer generated tree-like structures which may help to reveal the underlying behaviour of these Collatz sequences — which it must be emphasised are not one-one. In general we cannot retrace the history of a sequence as the inverse is often not unique — the tree branches!



Submissions should include program listings, hardware descriptions, run time and output; they will be judged for accuracy, originality and efficiency (not necessarily in that order). A prize of £10 will be awarded to the 'best' entry received by 1 September 1984. Please address entries to Mr MR Mudge, 'Square Acre', Stourbridge Road, Penn, Nr Wolverhampton, Staffs WV4 5NF.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided. Telephone comments, both favourable and otherwise, are welcome on (0902) 892141.

Review of Numbers Count -13- Kaprekar Numbers

In addition to numerous regular respondents both from the continent and the UK one submission introduced a new efficiency parameter; being the ratio of computer work time to programming time and estimated at 112hrs. : one evening using Knight's extended

This month's prizewinner is W J Duffin of 14 Orchard Croft, Cottingham, North Humberside, HU16 4HG using Basic on a BBC Micro 32k, OS 1.20, with a Microvitec medium resolution monitor, a Sanyo Slim 3A cassette recorder for saving programs and an MCP-40 printer/plotter or Olympia Compact 2 electronic typewriter for listing and output.

The exceptions in question A are seen to be the trivial case of all digits equal; also the case when the fourth differs by unity from the other three when the first difference is always 999.

It is hoped that the reproduction of Bill Duffin's results to questions B & C, together with his closing comments upon this problem, will encourage other readers to try their hand at 'Numbers'.

Due to lack of space, results to Question C are printed on page 209.

	Value of n							
Type of cycle *	2	3	4	5	6	7	8	9
C(1)		1 (954)	1 (7641)		2 (995544) (766431)		1 (98754210)	
C(2)				1 (95553)				
C(3)							1 (88763220)	
C(4)				2 (97641) (97443)				
C(5)	1 (81)							
(C7)					1 (875421)		1 (87664320)	
C(8)						1 (9775431)		
C(14)								1 (996654321)

Question B: tabulated results

For each cycle, one ordered n-digit number within the cycle is given, from which the others can be generated.

Thus the only self-generating numbers under this operation, having 9 digits or fewer, would seem to be:

495
6174
549945
631764
97508421

For example, C(3) for n=8 means that many 8-digit numbers eventually yield one of the differences 86526432, 64308654, 83208762 which regenerates.

NUMBERS

Factorials & primorials

Mike Mudge explores factorials and primorials which are near to prime.

Definitions

(i) A *prime number* is a positive integer which is divisible only by itself and unity. Thus the infinite sequence of prime numbers begins 2,3,5,7,11,13 . . .

(ii) The *factorial* of a positive integer, n, written n!, is defined to be the product of the positive integers less than or equal to n. Thus 6! = 1X2X3X4X5X6 = 720.

The sequence of factorials begins 1,2,6,24,120,720 . . .

(iii) The *primorial* of a prime number, p, written p*, is defined to be the product of the prime numbers less than or equal to p. Thus 7* = 2X3X5X7 = 210.

The sequence of primorials begins 2,6,30,210,2310 . . .

(iv) An integer, q, is said to be near-to-prime (NTP), if, and only if, either $q+1$ *or* $q-1$ are prime. (Note that if both $q+1$ and $q-1$ are prime then q is the mean of a prime pair; see Brun's Constant *PCW*, July).

Elementary Facts.

Factorial n is NTP for n=1,2,3,4,6,7 . . . since 2,3,(5,7), 23,719,5039 . . . are prime.

Primorial p is NTP for p=2,3,5,7,11 . . . since 3,(5,7), (29,31), 211,(2309, 2311) . . . are prime.

At least the first twenty-nine NTP factorials and the first seventeen NTP primorials are known; however, virtually nothing is known about their frequency of occurrence nor about their significance in analytic number theory.

Problem

Readers are invited to design and implement an algorithm for the determination of both NTP primorials and NTP factorials; attempting to reproduce and, if possible, extend the present results. Any possible suggestions as to the significance of these numbers would be most welcome.

Submissions should include program listings, hardware description, run time and output; they will be judged for accuracy, originality and efficiency (not necessarily in that order) and a prize of £10 will be awarded to the 'best' entry received by 1 December 1984. Please address submissions to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Nr Wolverhampton, Staffs WV4 5NF. Tel: (0902) 892141.

Review — Number Theories — March 1984

The original title was to have been Number Theory Nostalgia to emphasise the dates of the original solutions.

Submissions included the first Pro Pascal seen, from a Sirius 880 running at 5MHz; together with the expected Basic and Assembler programs running on NewBrain, Spectrum and BBC Model B computers.

(a) Complete solution *Math Quest Educ Times* vol 25 1876 p76. $(ax^5 \ldots dx^5)$, $a+b+c+d=x, x-a=p^5 \ldots x-d=s^5, x=(1/3)\ (p^5+\ \ldots\ +s^5)$ where $p=3m$, $q=3m+1$, $r=3m+2$, $s=3m+3$.

(b) *Amer Math Monthly* vol 2. 1895 pp128-9.

(c) *Amer Math Monthly* vol 5. 1890 p114 also vol 8. 1901 pp48-9. Consider the solution 3^5-D, 3^5, and 3^5+D . . .

(d) *L'intermédiaire des math* vol 11, 1904, pp16-7; the only known exception is 23.

(e) *L'intermédiaire des math* vol 24, 1917 pp23-41; the only known addition being $8191 = 1+2+\ \ldots\ +2^{12} = 1+90+90^2$.

This month's winner is John B Cook of 34 Joan Crescent, East Burwood, 3151 (232-2126), Australia, who used a TRS PC-2 with printer, as necessary.

John used 6.71 hours CPU time on (e) while Teilhet's limit of 600 in (d) was extended to 1800 in about 1½ days.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided. END

LEISURE LINES

by J J Clessa

Quickie

Divide 10 pounds of sugar into three portions so that three times the smallest portion equals the middle portion, and four times the middle portion equals the largest portion.

Prize Puzzle

A test of logic this month.

On the island of Nonesuch there are five species of birds:

— the Auk
— the Bluebird
— the Cockatoo
— the Drongo, and
— the Egret

Four birdwatchers, Peter, Quentin, Roger and Stanley, are located at different parts of the island when five different birds fly over in rapid succession. Each man makes his own identification of the birds, and the results are:

	Peter	Quentin	Roger	Stanley
Bird 1	Bluebird	Egret	Cockatoo	Egret
Bird 2	Auk	Auk	Bluebird	Auk
Bird 3	Cockatoo	Bluebird	Drongo	Cockatoo
Bird 4	Egret	Bluebird	Egret	Bluebird
Bird 5	Bluebird	Drongo	Auk	Drongo

In fact, none of the birdwatchers identified all the birds correctly, but conversely, no one had them all incorrect either. No two birdwatchers had the same numbers of incorrect guesses, and each of the five birds was correctly identified by at least one birdwatcher.

What were the five birds?

Answers, on postcards only, to *PCW* Prize Puzzle September 1984, Leisure Lines, 62 Oxford Street, London W1, to arrive not later than last post on 30 September 1984

June Prize Puzzle

A massive response to the June puzzle — almost 400 entries were received, most of them with the correct solution. The problem was easily solved by micro by testing all possible 6-digit square numbers for the required conditions. The required numbers (excluding solutions with leading zeros) are:

$494209=(703)^2$

and

$998001=(999)^2$

The winning entry drawn at random from the pile came from Dr David Vaux of the John Radcliffe hospital, Oxford. Congratulations Dr Vaux. Your prize is on its way. END

NUMBERS

Binomial coefficients

by Mike Mudge

The Binomial Coefficients, variously denoted by ${}^{n}C_{r}$ or $\binom{n}{r}$ are defined by $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ where integers n and r satisfy $0 \leqslant r \leqslant n$; and $r! = 1.2.3....r, r>0$; $0! = 1$. For example. $\binom{20}{10} = 184756$.

These coefficients, whose common occurrences include the algebraic result $(A+B)^n = \sum^{n}\binom{n}{r}A^r B^{n-r}$ together with The Bernoulli Distribution in applied statistics are directly available (for sufficiently small n and r) on many scientific calculators.

However, since $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$ the Binomial Coefficients may be written down using Pascal's Triangle. Thus:

n=1	1 1
n=2	1 2 1
n=3	1 3 3 1
n=4	1 4 6 4 1

....

The array is edged with ones, while each interior term is the sum of the two terms immediately above it; r is counted along the diagonals of the array.

Let's turn our attention to certain results concerning the number theoretic properties of the $\binom{n}{r}$ as distinct from their usage in other branches of mathematics.

(1) If the binomial coefficients are factorised thus: $\binom{n}{k} = UV$ where every factor of U is less than or equal to k while every factor of V is greater than k there are known to be finitely many coefficients with $n \geqslant 2k$ for which $U>V$. Determine such cases for $k = 3,5,7,....$ *Note.* If k=3 then n= 8,9,10,18,82 and 162 while for k=5, n= 10, 12, and 28 are known to be the only cases for $n \leqslant 551$.

(2) If n is a prime number greater than 3, the Wolstenholmes' Theorem states that $\binom{2n-1}{n}$ is congruent to 1 modulo n^3: that is $\binom{2n-1}{n} = An^3 + 1$, where A is an integer. Display empirical evidence for this theorem and consider the possible truth of its converse.

(3) What can be said about the largest divisor of $\binom{n}{k}$ which is less than n?

If $n \geqslant k^2 - 1$ there is a prime divisor less than or equal to n/k apart from $\binom{62}{6} = 61474519$.

If $n<k(k+3)$ there is a prime divisor less than or equal to $k+1$ apart from the cases $\binom{7}{3}$, $\binom{14}{9}$, $\binom{23}{5}$, $\binom{44}{8}$, and $\binom{47}{11}$.

How do we best generate such divisors?

(4) When is$\left(\binom{2n}{n}, 105\right) = 1$? That is, when do $\binom{2n}{n}$, and 105 have no common factor. RL Graham offered a prize of \$100 for proving that this happened infinitely often. At least fourteen values of n are known n = 1,10,756,...

If g(n) is the smallest prime factor of $\binom{2n}{n}$, then $g(3160) = 13$ and $g(n) \leqslant 11$ for $3160 < n < 10^{110}$. Does this help?

Readers are invited to investigate the above problems and results with an objective of extending empirical evidence and possibly generating new conjectures.

Submissions should include program listings, hardware description, run times and output. These will be judged for accuracy, originality and efficiency (not necessarily in that order) and a prize of £10 will be awarded to the 'best' entry received by 1 January 1985.

Please address submissions to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Nr Wolverhampton, Staffs.

Review—Repunits—April 1984

Repunits have found favour far and wide: a most elegant work, conjecturing the primality of $aR_k + b$ when

a	b	k
3	−2	50&60
5	−2	66
7	2	66

and establishing the primality of $6R_{66} + 1$ in less than three hours in Forth on a Jupiter Ace arrived from Denmark.

A Sharp PC3201 using Z80 machine code on The Isle of Mull suggests that a prime p greater than 3 will be a factor of $R_{n(p-1)}$ for n=2,3,...

Substantial references on the subject of repunits are to be found in the paperback *Repunits and Repetends* by Samuel Yates published by The Star Publishing Company, Boynton Beach Florida 1982.

This month's winner is Robin Merson of 2 Vine Close, Wrecclesham, Farnham, Surrey, GU10 4TE. Robin appeals to any reader who has implemented the Primality Testing Algorithm described by H Cohen & HW Lenstra (*Mathematics of Computation*, vol.42. Jan 1984) to contact him on Frensham 3587 for an 'info' exchange with Hardy and Wright-type algorithms.

A summary of Robin's results follows. Further details are available from him or myself.

Using an Apple microcomputer several primes have been found of the form d(n)e, where d=5,6,7 or 8 and e has one or two digits. An embedded (n) in a number implies the previous digit is repeated n times. For example, 10(4)2 stands for 100002. The largest for each of the values of d are 5(92)21, 6(120)1, 7(99)1 and 8(138)1.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided. END

LEISURE LINES

by J J Clessa

Quickie

Here is a remark attributed to that famous geometrician, Pl Thagorus: 'Now I have a rough predictor of circle areas and volume.'

Prize Puzzle

And now one for the micros.

In the annual festival games held at the village of Little Dingblat in West Sussex, the main event is the marathon.

This year, all the entrants were numbered sequentially (1, 2, 3 . . .).

By coincidence, all those who completed the race carried numbers which were either exact primes or exact powers of other numbers. Furthermore, the total of the race numbers of the finishers was exactly equal to the total of the race numbers of those who dropped out.

How many entrants were in the race?

Answers, on postcards only, to Leisure Lines, *PCW* Prize Puzzle, October 1984, 62 Oxford Street, London W1, to arrive not later than 31 October 1984.

July Prize Puzzle

The problem was quite easily cracked by micro and several people sent in their programs and printouts.

The winning entry came from Adam Jefferson of Bradford, Yorks. Congratulations, Adam, your prize is on its way.

The solution to the problem was that the Dawsons and the Firths were the R C families—the children of these families were given £31.25 each.

Keep puzzling.

Oct 1984

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NUMBERS

Triperfect numbers

Mathematical mind-benders from Mike Mudge

A positive number N is called a 'triperfect' number if and only if s(N)=3N, where s(N) denotes the sum of the positive divisors of N.

For example: $T_1=2^3.3.5=120$ is triperfect because 1+2+3+5+4+6+10+15+8+12+20+30+24+40+60+120 =3.120 =360.

And: $T_2=2^5.3.7=672$ is triperfect because 1+2+3+7+4+6+14+21+8+12+28+42+16+24+56+84+168+112+48+32+336+96+224+672=3.672=2016.

We need not search for odd triperfect numbers since it has been shown that such numbers (if they exist) are:
(i) greater than 10^{50}, WE Beck & RM Najar, *Math Comp, vol 38, 1982, pp249-251.*
(ii) mutiples of at least 10 distinct prime factors, M Kishore *Math Comp, vol 42, 1984, pp231-233.*

However, six even triperfect numbers are known (including T_1 & T_2 above)

Readers are invited to design and implement an algorithm for the determination of some or all of these numbers.

Submissions should include program listings, hardware description, run times and output; these will be judged for accuracy, originality and efficiency (not necessarily in that order) and a prize of £10 will be awarded to the 'best' entry received by 1 February 1985.

Please address entries to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Nr Wolverhampton, Staffs WV4 5NF. Tel: (0902) 892141.

Diophantine equations

The response to this three-part problem displayed the considerable interest in the solution of equations in integers, both by computational and algebraic methods.

The wording of (1) was unfortunately somewhat misleading and prompted numerous telephone calls and letters; copies of the Barnes paper are available on request so that the detailed nature of the 'trivial' solutions may be seen.

The fourth non-trivial solution to (2) x=74, y=260 together with a further solution x=767, y=8672 hitherto unknown to the writer have been found. Are there any others?

In (3) the value x=3344161 was incorrectly printed but this did not deter extensive algebraic and computational analysis.

The May prize-winner is AS Tickner of 14 Grimsdyke Road, Pinner, Middlesex HA5 4PH.

The criteria used to justify the award for work on an HP-98020 A calculator are those listed together with a commendable tenacity of purpose. It is hoped that other contributors and those interested in Diophantine problems will contact Mr Tickner in an attempt to complete the analysis of problem (2) and to investigate further unsolved problems in this area.

Quotients of Wilson and Fermat

This proved to be a very popular competition, probably due to the simplicity of its formulation. A ZX81 ran for 32 days, a U2200 (Apple-compatible) in Pascal used 36-digit LONGINT to display explicit values of some F_p and W_p, while an Apple II in 14 days 3 hours and 22 minutes made a major contribution. However, several programmers were in difficulty with unrecognised integer overflow leading to incorrect solutions of $w_p=0(p)$.

Nonetheless, we can now list all the solutions of $a^{p-1}\equiv 1 \pmod{p^2}$ for $2\leqslant a\leqslant 31$ and $3\leqslant p\leqslant 1040069$.

a	p
2	1093,3511
3	11,1006003
5	20771,40487
6	66161,534851
7	5,491531
10	3,487
11	71
a	p
12	2693,123653
13	863
14	29,353
15	29131
17	3,46021,48947
18	5,7,37,331,33923
19	3,7,13,43,137
a	p
20	281,46457
22	13,673
23	13
24	5,25633
26	3,5,71
28	3,19,23
30	7,160541
31	7,79,6451

(i) No solutions were found for a=21 or a=29.
(ii) There are no further solutions for a=2 with p less than 3×10^7.

Considering $(p-1)!=-1 \pmod{p^2}$. EH Pearson, *Mathematics of Computation, vol 17, 1963, p194* has tested all p less than or equal to 200183 and only p=5,13 and 563 satisfy the congruence . . .

The June prize-winner is Don Hunter of The Old Vicarage, Elmdon, Nr Saffron Walden, Essex CB11 4LT; whose ALGOL 60 programming of a 16k Elliott 903 using library procedures for multi-precision arithmetic ran for a total of 53½ hours.

The last such computer that the writer heard of had been refurbished following its removal from a tank.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided. END

LEISURE LINES

Brain-teasers courtesy of J J Clessa

Quickie

What temperature is the same in Centigrade (Celsius) as it is in Fahrenheit?

Prize Puzzle

A certain 7-digit number contains no zeros and is not palindromic (that is, it does not read the same from right to left), but it does have the property that if its digits are reversed, the resulting 7-digit number is a factor of the original number. What is the original number?

Answers, on postcards only, to: *PCW* Prize Puzzle, November 1984, Leisure Lines, 62 Oxford Street, London, W1. Entries to arrive not later than 30 November 1984. END

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NUMBERS

Mathematical mind-benders from Mike Mudge

This month sees a dramatic change in the area of research covered in Numbers, a move from integer arithmetic to floating point arithmetic.

Consider the decimal expansion of a fraction (which for convenience will be supposed to lie between 0 and 1). This either terminates, for example, 73/200 = .365 or yields a repeating pattern called a recurring decimal, say, 7/13 = .538461 538461 538461 . . . written .538461. There is little of interest for us in such cases.

However, suppose that we start with an irrational number, which by definition cannot be exactly represented as a fraction. What happens in its decimal expansion?

A number is said to be *simply normal* if each of the digits 0, 1, . . .9 occur equally often in the non-terminating expansion as a decimal; furthermore, it is said to be *normal* if every combination of these digits occurs with the proper frequency — by which we mean the frequency calculated on the assumption of randomness . . . the absence of any pattern.

A number which preserves the property of normality in every possible number system, including of course binary, is said to be *absolutely normal.*

Now we shall restrict our discussion to two famous irrational numbers:

(1) 'Pi', π, the ratio of the circumference of a circle to its diameter.

Readers may wish to use the series: $\pi = 1/1\ (16/5 - 4/239) - 1/3(16/5^3 - 4/239^3) + 1/5(16/5^5 - 4/239^5) \ldots$ approximately 3.1415926536.

(2) 'e', the base of natural logarithms, defined by the series:

$e = 1 + 1/1 + 1/(1.2) + 1/(1.2.3) + 1/(1.2.3.4) + 1/(1.2.3.4.5) \ldots$ approximately 2.7182818285.

Readers are encouraged to examine and improve upon the very crude ZX81 Basic program given here.

```
1 DIM X(100)
2 DIM Y (10)
3 DIM Z (175)
4 DIM W (60)
5 FAST
10 LET N = 40
15 PRINT "THE CALCULATION OF E TO ";N," DECIMAL PLACES YIELDS . . ."
20 LET M = 4
22 FOR D = 1 TO 10
23 LET Y (D) = 0
24 NEXT D
30 LET TE = (N + 1)* 2.30258509
33 LET M = M + 1
34 LET DI = M* (LN (M) - 1) + 0.5 × LN(6.2831852* M)
35 IF DI <= TE THEN GOTO 33
36 PRINT " AN M REQUEST OF ";M
38 FOR J = 2 TO M
40 LET X (J) = 1
42 NEXT J
44 LET FI = 2
45 LET Y(2) = 1
48 FOR I = 1 TO N
50 LET CA = 0
52 LET J = H
54 LET TE = X(J)* 10 + CA
56 LET CA = INT (TE/J)
58 LET X(J) = TE - CA* J
60 LET J = J - 1
62 IF J >= 2 THEN GOTO 54
64 IF CA - 0 THEN GOTO 68
66 GOTO 70
68 LET Y(10) = Y(10) + 1
69 GOTO 72
70 LET Y(CA) = Y(CA) + 1
72 LET Z(I) = CA
74 NEXT I
76 PRINT FI; ".";
78 FOR Q = 1 TO 60
80 PRINT Z(Q);
81 NEXT Q
82 LET J = 61
84 IF J >= N THEN GOTO 97
86 LET M=J-1
88 FOR D = 1 TO 60
89 LET M = M + 1
90 LET W(D) = Z(M)
91 NEXT D
92 FOR E = 1TO 60
93 PRINT W(E);
94 NEXT E
95 LET J = J + 60
96 GOTO 84
97 PRINT
98 PRINT "END"
```

With each of the numbers π and e (and if a further challenge is needed the square root and cube root of 2), readers are invited to submit programs to calculate any required number of decimal places and to test at least for simple normality by counting the numbers of each digit present in the resulting decimal expansion.

Test Data: computed on ENIAC around 1950 took approximately 11 hours for e with a further 17 hours for card-handling and checking, and a total of around 70 hours machine running time for π. Readers will appreciate how computing has changed over the past quarter of a century (see Fig 1).

Submissions should include program listings, hardware description, run times and output; these will be judged for accuracy, originality and efficiency (not necessarily in that order) and a prize of not less than £10 will be awarded to the 'best' entry received by 1 March 1985.

Please address entries to Mike Mudge, 'Square Acre', Stourbridge Road, Penn, Nr Wolverhampton, Staffs. WV4 5NF. Tel: (0902) 891141.

Please note that submissions can only be returned if a suitable stamped addressed envelope is provided. Expanded reviews of previous 'Numbers' problems together with, subject to the approval of the contributor, copies of detailed programs from the prize-winning submission may also be requested.

END

Digit	0	1	2	3	4	5	6	7	8	9
2,035 decimals of π	184	213	210	190	198	211	204	200	207	218
2,000 decimals of e	196	190	208	202	201	197	204	198	202	202

Fig 1

LEISURE LINES

Brain-teasers courtesy of J J Clessa

Quickie

Their are three mistakes in this sentense — can you find them?

Prize Puzzle

This one should keep the micros humming over the Christmas period. I believe the puzzle was originated by Ernest Dudeney but it's certainly been around a while.

Can you find an integer which — divided by 5 and multiplied by 4 — gives the same result if you move the first digit of the number to the end.

For example, suppose the number is 2615. If you divide it by 5 and multiply it by 4 you get 2092. But if it were the number we were seeking, we would get 6152.

Answers please, on postcards only, to: *PCW* Prize Puzzle, December 1984, Leisure Lines, 62 Oxford Street, London W1. Entries to arrive not later than 31 December 1984.

August Prize Puzzle

The answers are as follows:

(a) The largest perfect square with digits in ascending order is 134 689.

(b) The largest perfect square with digits in descending order is 961.

Winner: D Haworth, Bolton, Lancs. Congratulations!

END